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REDETERMINATION OF THE WAVE-LENGTHS OF THE ARC AND THE SPARK LINES OF TITANIUM, MANGANESE, AND VANADIUM; THE EFFECT OF CAPACITY AND SELF-INDUCTION ON THE WAVE-LENGTHS OF THE SPARK LINES

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I. WAVE-LENGTHS OF ARC AND SPARK LINES

INTRODUCTION

The importance of accurate determinations of the wave-lengths is of course fully recognized, and so this undertaking needs no apology. The history of such work is parallel with that of spectroscopic instruments. With each improvement of instruments greater accuracy has been attained, and finally such accuracy as to cause perception of the difference between the solar and the arc standards of wave-length.

As the iron arc standards have now been generally accepted, the necessity of redetermining the wave-lengths formerly based on solar standards is apparent. This fact, together with the opportunity afforded by the accessibility to the most excellent 21.5-ft. Rowland concave grating with 20,000 lines per inch, suggested the undertaking of this work.

ARC SPECTRA

Apparatus and methods.—The mounting, adjustment, and theory of the grating have been described by Professor Ames.[1] The source of light used for the arc spectra consisted of an arc produced by 110

[1] *Phil. Mag.*, **27**, 369, 1889.

volts and 11 amperes between carbon terminals, the positive being loaded with the substance under examination. The steadiest and most satisfactory arc was obtained by using equal parts of the metal and carbon dust with the upper pole positive. The arc-stand was rigidly fixed in position at a distance of 125 cm from the slit and in line with the slit and the grating. The light was focused on the slit by a movable quartz lens supported by a tube which was rigidly fixed in line with the slit and the grating.

The iron arc for comparison was a 3-ampere arc such as has been described by Pfund.[1] The terminals were placed in the same stand used for the arc of the other substance.

By means of a shutter with a horizontal slot of the same width as the thickness of the shutter and on a horizontal axis, the spectrum of the substance was taken on the center of the plate and the comparison spectrum on the outer portions.

As mechanical shifts were found on many of the plates several methods of correctly determining the true relative positions of the lines of the two spectra were used. By taking plates of an arc between a positive pole of the substance and a negative pole of iron, the true relative positions of the lines could be determined. When the impurities common to the two arcs were sufficiently strong, the mechanical shift was at once apparent. Another, and more satisfactory, method was to load the positive carbon pole with the substance, iron-filings, and carbon dust, intimately mixed in the ratio 1:1:2. In many cases all three methods were used. The last scheme makes it unnecessary to use the iron arc for comparison, besides obviating mechanical shifts. To test whether the iron lines obtained by using the mixture suffered any displacement because of the presence of the other substances, exposures were made giving the arc of the mixture on the center of the plate and the standard iron arc on either side. The coincidence of the ends of the lines showed that no displacement had occurred.

SPARK SPECTRA

Apparatus and method.—The spark used was produced by a 110-volt alternating current of 25 amperes with a frequency of 60 cycles per second. By means of a transformer the voltage was "stepped up"

[1] *Astrophysical Journal*, **27**, 296, 1908.

to 1000, which was again "stepped up" to about 20,000 by another transformer. This potential was sufficient to produce a spark of about 2 cm in length. The spark terminals were of conical-shaped metal, which was reshaped for each exposure in order to keep the spark steady.

In a circuit shunting the spark were condensers consisting of copper foil and glass surrounded by moisture-free transformer oil and having a capacity of 0.03 micro-farad.

The spark used was about 8 mm in length, and its image on the slit was about 7 mm. The capacity that gave the best sparking results when no self-induction was introduced was 0.016 micro-farad, which amount remained constant throughout the work on the wave-lengths of the spark lines.

The comparison arc consisted of loaded carbon poles as described above. The terminals of both arc and spark were fixed in exactly similar sliding clamp-rods, having but one degree of freedom, which were inserted in the same sockets of the fixed arc-stand. Marks on the rods insured the same positions of arc and spark relative to the slit. Moreover, as the focusing lens remained in position after one exposure any difference in the positions of the two sources was at once detected by the displacement of the image on the slit. Though care was taken to have the positions the same, tests, which will be discussed later, proved that great caution was unnecessary.

To avoid mechanical shifts as far as possible, the shutter of the camera-box was detached and fastened to clamp-stands resting on the floor, and the plate-holder was wedged. As in the case of the arc spectra the shift could be detected by comparing the lines in the two spectra due to impurities. But to avoid any possibility of error due to a mechanical shift the method of half-exposure was used, i. e., the plate was exposed half-time to the spark, full time to the arc, and then half-time again to the spark.

The exposures were made in the second-order spectrum, in which 1 Å. U. is approximately equal to 1 mm on the plate. Seed's "Gilt Edge" No. 27, Cramer's "Isochromatic," and Wratten and Wainwright's "Panchromatic" plates were used. As the Gilt Edge plates are not sensitive above λ 5100, the only screens necessary were plate glass and picric acid.

MEASUREMENTS

Method.—In calculating the wave-lengths, the standard iron arc lines as determined by Buisson and Fabry[1] were used. The measurements were made with the dividing-engine constructed by Rowland especially for this work and by the method described by Humphreys.[2] After reducing the scale of the engine to Ångström units by multiplying by a number slightly different from unity, corrections were made from a calibration-curve for each plate, which was drawn with the standard wave-lengths as abscissae and the differences between these wave-lengths and the readings of the same lines as ordinates. The corrections for each line could then be read from the curve.

As a check on the work the plates were taken so as to overlap.

In estimating the intensities, the iron arc line λ 4045.969 was used as the standard, and its intensity is marked 10, the same as that assigned by Kayser.

Symbols.—*n* indicates that the line is ill-defined or nebulous; n^r that it is nebulous on the less refracted side; n^b that it is nebulous on the more refracted side; and *r* that it is often reversed.

[1] *Comptes Rendus*, **144**, 1155, 1907; *Journal de Physique* (4), **7**, 169, 1908; *Astrophysical Journal*, **28**, 169, 1908.

[2] *Astrophysical Journal*, **6**, 180, 1897.

TABLE I

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
2802.465	8	7	3056.736	5	4
05.680	5	5	58.075	8	7
06.407	2	7	59.731	8	7
09.150	4	3	60.452	2	1
10.276	4	$8n^r$	63.484	3	2
12.963	3	3	66.197	8	7
17.838	3	$8n^r$	66.357	6	5
20.249	3	5	66.515	5	4
25.377	3	3	71.229	5	5
28.150	3	$7n^r$	72.099	8	7
32.158	8	10	72.971	10	8
41.914	8	10	75.220	10	$8n$
51.087	5	8	78.638	10	$8n$
53.922	4	7	80.172	3	1
58.399	3	6	84.812	2	1
61.291	3	4	88.026	10	$8n$
62.213	6	8	89.394	6	$6n$
68.732	4	7	90.127	5	1
77.418	7	8	97.176	8	7
84.099	8	$8n^r$	3101.511	2	2
86.036	3	3	03.800	7	6
87.456	2	6	05.080	7	5
88.923	5	7	06.225	5	6
91.050	5	7	06.800	5	1
2905.649	4	4	07.453	4	1
09.912	4	3	09.576	2	1
12.072	7	6	10.674	7	4
23.484	3	2	11.278	2	1
24.005	2	4	12.050	4	3
28.320	8	6	12.478	5	1
33.526	8	4	14.092	6	
37.301	8	4	17.454	3	
41.993	10	$9n^r$	17.663	5	6
46.784	10	$9n^r$	18.130	4	
48.247	8	7	19.745	9	7
56.130	8	6	21.595	3	1
56.796	10	4	23.074	7	1
65.681	10	5	23.764	6	1
67.220	10	4	28.640	3	1
68.231	4	3	29.069	3	1
70.376	6	3	30.790	8	7
70.552	6	3	35.926	2	
74.926	4	3	41.513	6	1
83.203	8	4	41.673	5	1
3000.866	7	7	43.750	6	5
02.727	6	6	44.718	3	1
17.175	8	$10n^r$	47.258	2	
29.723	4	10	48.030	8	8
42.542	3	1	52.239	8	7
43.847	3	3	53.591	3	
46.671	7	7	54.184	8	7
48.766	4	3	55.654	8	$6n^r$

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3157.390	4	2	3251.890	10	8
61.190	8	8	52.849	10	10
61.760	8	8	54.232	10	9
62.557	8	8*nr*	60.256	2	2
68.519	9	8*nr*	61.587	10	18
70.923	2	1	63.676	2	3
72.721	3	1	70.562	1	1
79.284	3		71.631	8	8
83.970	4	1	72.064	8	8
86.454	8	5	75.283	2	3
90.868	7	7*n*	76.759	5	4
92.003	8	4	78.276	8	9
95.707	4	3	78.908	8	9
97.508	4	3	79.976	3	4
99.908	8	6	82.320	7	8
3201.586	3	1	87.642	7	10
02.524	6	6*n*	88.125	3	1
03.420	4	2	88.418	2	2
03.815	6	1	88.578	3	4
04.866	4	1	92.059	8	4
05.845	4	1	94.889	2	1
06.340	4	1	99.455	4	3
06.820	4	1	3302.093	2	2
07.328	4	1	06.872	4	1
07.896	4	1	08.382	4	1
09.021	4	1	08.794	6	7
13.136	6	2	09.715	3	2
14.227	8	1	12.679	3	1
14.756	8	2	14.403	6	3
17.042	9	9*n*	15.320	4	8
17.946	3	1	18.006	9	9
18.253	6	8*n*	18.351	2	1
19.208	3	1	21.576	3	3
21.374	3	1	21.700	4	4
22.823	10	8*n*	22.934	10	10
24.227	3	7*n*	26.757	8	8
26.114	5	1	29.450	10	10
26.757	4	3	32.101	8	8
28.586	8	8	35.188	10	10
29.181	8	7	37.892	3	3
29.404	8	7	40.333	8	8
31.299	6	3	41.869	10	9
32.263	7	7	42.137	5	3
34.517	10*n*	12*n*	42.700	2	1
36.102	6	7	43.757	8	6
36.568	9	10	46.724	9	6
38.203	2	1	48.821	8	7
39.030	9	8	49.022	8	7
39.646	8	7	49.408	10	8
41.970	10	12	52.059	3	3
48.600	8	9	52.919	4	1
49.368	2	3	54.642	9	5

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3358.263	8	3	3481.683	1	
58.467	2	1	85.686	2	1
61.218	10*n*	10	89.735	5	4
61.821	5	1	91.047	9	7
62.099	2	1	93.270	4	1
66.171	5	4	95.734	4	1
67.874	2	1	99.098	5	1
69.196	3	3	3500.333	4	2
70.423	8	5	04.887	10	15
71.442	10	5	06.646	3	1
72.197	7	4	07.433	2	1
72.804	10	10	10.840	8	15
74.340	3	3	16.842	2	1
79.200	7	8	20.246	8	9
80.279	8	10	25.241	2	
82.300	6	2	30.578	2	
83.759	10n^r	10*n*	35.395	8	10
85.934	8	5	37.472	2	1
87.830	9	8	47.011	5	1
88.747	3	4	66.095	3	2
90.668	3	1	73.716	5	4
92.199	3	2	74.241	2	1
94.567	9	10	85.861	3	1
98.620	2	1	87.132	2	4
3402.416	4	4	96.046	7	7
03.358	4	1	98.710	6	1
05.085	4	1	3604.295	2	1
07.196	5	4	06.057	1	
09.801	5	3	07.122	2	
11.670	2		12.253	1	
15.985	3		13.749	2	
16.954	3	2	14.195	3	
28.953	2	1	16.957	2	
39.300	2	1	19.453	2	1
43.639	2	2	22.998	4	1
44.326	9n^r	8n^r	24.843	5	7
52.480	3	5	26.087	2	1
55.392	2		26.923	1	1
56.395	5	5	35.198	3	1
56.657	3	1	35.469	8n^r	7
57.490	2		37.965	3	1
59.430	2	1	41.327	4	7
61.506	10n^r	9n^r	42.680	10	9
63.230	2	1	44.461	2	1
65.640	3	4	44.692	2	1
67.268	4	1	46.191	3	2
76.450	1	1	53.493	8*n*	8
76.986	3	1	54.577	4	2
77.184	8	9	58.140	6	4
78.921	4	1	59.748	5	7
80.533	7	2	60.619	5	2
80.891	2	1	62.222	5	5

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3668.950	5	2	3836.511	2	1
71.662	5	2	41.747	3	1
79.680	2	2	75.286	4	2
85.185	10*n*	15	84.035	2	1
85.949	3	2	88.029	2	1
89.892	5	2	89.956	3	1
94.429	4	1	93.635	1	1
96.882	2		95.238	5	2
98.174	3	1	97.315	2	2
3700.047	3	1	97.578	2	2
01.527	1		98.489	5	2
02.286	4	1	3900.531	7	10
02.976	2		00.957	5	2
04.285	4	2	04.769	8	5
06.215	4	5	11.182	3	2
07.525	2	1	12.575	2	1
08.645	2	1	13.452	8	10
09.951	4	1	14.328	7	3
15.329	2		14.720	3	1
17.391	5	3	15.875	2	2
19.931	3	2	19.815	3	1
21.636	4	5	21.418	5	2
25.119	4	2	24.515	8	4
29.767	8*nr*	5	26.322	4	2
33.782	2	1	27.330	2	1
38.901	3	1	29.865	6	3
41.060	6	4	32.017	3	3
41.140	5	8	34.230	3	3
52.867	8	5	47.754	8	3
53.632	5	3	48.664	10	4
57.683	3	5	56.280	10	5
59.296	10	15	58.208	10	5
61.325	9	12	62.861	7	2
62.326	3	3	64.267	7	3
66.461	2	1	81.766	8	5
69.758	2	1	82.540	5	3
71.642	4	3	84.331	1	1
74.633	2	1	87.756	1	1
82.139	2	1	89.767	10	5
86.025	4	4	92.657	2	1
86.274	2	1	94.696	2	1
3801.065	2	2	98.645	10	6
05.134	2	1	99.466	2	1
05.478	2	1	4002.495	3	1
11.333	5	1	05.979	3	1
13.392	3	3	07.212	1	1
14.860	3	3	08.073	3	1
18.215	3	2	09.137	7	3
21.740	4	2	09.677	4	2
28.174	2	1	12.403	4	3
29.410	3	2	12.813	1	1
36.103	4	2	13.600	4	2

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
4015.398	4	2	4150.549	1	2
16.304	2	2	50.967	4	3
16.988	1	1	58.055	2	1
17.781	4	1	63.653	4	10n^r
24.572	8	3	69.330	2	1
25.130	3	2	71.023	2	
26.545	5	2	71.910	2	8n^r
27.614	2	1	73.797	3	3
28.352	4	4	74.471	1	2
30.527	4	2	80.864	3	1
31.773	2	1	83.287	2	1
32.600	1	1	86.118	4	3
33.922	2	1	88.685	1	1
34.918	2	1	4200.786	2	2
35.841	3	1	03.786	4	2
38.322	1	1	11.723	2	1
40.326	2	1	15.520	2	1
43.768	1	1	24.784	2	1
49.392	2	1	27.641	2	1
52.948	1	1	37.883	4	4
53.835	3	4	45.513	2	1
55.021	4	2	49.126	3	1
57.640	3	2	51.604	2	1
58.154	4	4	51.756	1	1
60.269	4	3	56.039	4	2
64.215	4	2	58.531	3	1
65.106	4	2	61.611	3	1
68.140	1	1	63.142	5	5
68.995	1	1	65.712	2	1
70.212	1	1	66.205	2	1
71.476	1	2	70.145	2	1
77.153	2	1	72.445	4	3
78.471	6	4	73.309	2	1
79.730	3	1	74.600	5	5
82.458	4	3	76.443	4	3
99.167	3	1	76.658	2	4
4100.936	2	1	78.135	3	3
02.716	1	1	78.815	3	1
12.726	4	3	80.080	2	1
21.648	2	1	80.343	2	1
22.167	3	2	81.650	4	3
23.308	2	1	82.715	5	3
23.381	3	1	84.990	4	5
27.544	4	3	86.012	8	6
27.968	1	1	87.412	8	6
29.172	2	2	88.181	3	4
31.257	2	1	89.080	6	5
37.290	4	2	89.922	3	6
42.493	2	1	90.228	2	9n^r
43.044	3	1	90.936	4	4
43.280	2	1	91.136	4	3
48.486	1	1	94.102	6	8

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
4295.755	5	6	4416.535	4	2
98.666	3	6	17.278	5	4
99.140	4	4	17.712	4	5
99.641	5	4	21.467	2	1
4300.047	4	9	21.753	4	3
00.555	4	5	22.822	4	3
01.082	4	6	24.399	2	1
01.934	4	5	25.831	1	1
05.907	4	10	26.053	4	3
07.885	3	10	27.102	5	6
09.088	1	1	30.034	4	1
10.361	1	1	30.367	3	3
11.648	2	2	31.282	3	3
12.875	4	8	32.601	3	2
14.341	3	6	33.581	2	3
14.800	3	6	33.993	5	4
18.635	5	3	36.592	4	3
20.949	3	2	38.233	2	2
21.659	4	2	40.343	4	4
23.435	2	1	41.273	3	3
25.130	5	3	43.201	3	1
26.351	4	3	43.801	4	1
26.983	2	3	44.262	5	12
34.856	3	1	49.127	6	9
37.915	7	8	49.991	2	1
38.498	2	1	50.493	4	6
40.062	1	1	50.902	6	6
41.391	3	3	53.323	6	6
44.314	3	4	53.697	5	5
46.127	4	2	55.324	9	6
46.598	3	1	57.427	9	8
54.080	3	1	62.084	2	1
55.323	2	1	63.387	3	3
60.510	4	2	63.552	3	3
61.151	1	1	64.465	3	4
68.974	2	1	65.803	6	5
69.698	5	3	68.493	6	12
72.403	3	2	70.876	2	4
75.435	1	1	71.235	5	5
88.085	2	2	74.856	4	3
93.927	5	4	79.700	5	3
95.037	10	12	80.607	4	2
95.860	3	3	81.267	6	6
99.772	4	6	82.693	5	3
4400.599	2	1	88.315	3	7
04.273	5	4	89.096	6	5
05.696	2	1	92.557	2	1
07.667	3	1	95.018	4	1
08.199	2	1	96.150	5	4
08.393	3	1	97.747	2	1
12.437	2	1	4501.271	6	11
15.574	1	1	03.777	3	1

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
4506.358	2	1	4667.589	9	8
08.044	1	1	75.125	4	3
08.280	1	1	81.914	8	8
11.176	3	2	90.810	3	1
12.736	8	8	91.341	5	5
18.026	8	8	93.682	3	2
18.691	4	3	96.947	3	1
22.303	8	8	98.790	5	5
27.315	6	7	4710.194	5	4
33.246	9	9	15.312	3	1
33.968	4	9n^r	22.621	4	2
34.777	9	8	23.179	4	2
35.575	4	8	31.178	4	2
35.923	6	6	33.436	3	2
36.001	5	6	34.684	2	1
37.232	2	2	42.126	2	1
40.870	1	1	42.801	5	5
44.700	8	8	47.694	2	2
48.772	8	8	58.134	8	8
49.635	8	15	58.919	2	2
52.563	8	8	59.283	7	8
55.104	2	1	66.331	2	2
55.498	7	8	69.784	2	2
58.122	2	1	78.271	3	3
59.937	4	1	81.726	3	2
62.630	3		92.499	4	3
63.436	2	2	96.223	3	2
63.765	4	8	97.994	2	2
64.229	1		99.817	4	4
70.911	3	3	4805.109	3	7
71.981	8	15	05.434	4	4
75.526	2	1	08.536	3	3
85.839	2	3	11.087	2	2
89.962	4	4	12.248	3	2
99.232	4	2	19.043	1	2
4609.375	3	2	20.420	4	4
14.292	1	1	27.597	1	2
17.268	9	11	36.128	2	2
19.532	2	1	40.878	5	5
23.098	6	8	43.990	1	2
29.335	6	6	48.460	3	3
34.878	2	2	56.013	5	6
35.548	2	1	68.286	4	4
37.875	2	2	70.141	6	6
39.361	4	4	80.918	1	1
39.665	4	4	82.342	1	2
39.945	4	4	85.093	6	8
45.188	4	3	93.063	1	1
50.017	4	3	93.433	1	1
55.704	2	2	99.925	6	7
56.060	3	2	4911.188	1	3
56.474	7	6	13.625	4	6

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
4915.241	2	2	5206.089	2	2
19.879	1	3	10.394	6	8
21.785	3	3	19.720	2	6
25.422	2	2	22.683	2	4
26.170	1	2	23.636	2	2
28.357	3	3	24.323	4	8
37.746	1	1	24.960	4	6
38.306	3	2	26.555	2	2
41.583	1	1	27.187	2	2
48.214	1	1	38.575	2	4
64.750	2	2	46.740	2	4
68.491	2	2	46.811	2	2
73.055	2	4	52.107	2	4
75.358	3	2	55.824	2	4
77.748	3	4	59.985	2	4
78.209	3	2	63.316	2	2
81.746	10	11	63.504	2	2
89.161	3	3	65.968	4	8
91.077	9	10	66.281	2	2
97.100	2	2	81.928	2	4
99.512	9	9	82.376	2	6
5001.011	3	3	83.448	4	8
07.218	8	8	84.389	2	4
09.650	3	3	89.100	2	2
13.302	4	4	89.278	2	2
14.262	8	7	89.731	2	2
16.174	4	7	94.754	2	2
20.045	5	7	95.783	4	3
22.872	4	6	97.293	4	7
24.853	4	5	98.423	4	6
25.580	4	6	99.828	2	3
35.916	4	6	5335.348	2	2
36.474	4	6	51.094	4	5
38.412	4	5	69.653	4	6
39.962	4	5	90.004	2	3
40.627	2	2	90.259	2	3
43.590	2	3	5409.625	4	6
45.430	2	2	26.268	2	4
52.880	2	4	29.155	3	6
62.115	2	4	36.651	2	2
64.659	5	7	38.327	2	4
66.007	2	3	46.671	2	4
71.490	2	2	49.187	2	3
87.069	2	2	53.672	2	4
5113.441	4	6	60.614	6	5
20.429	4	6	71.188	4	4
45.470	4	4	74.284	4	7
47.476	4	4	77.732	4	5
52.184	4	5	81.450	4	6
73.744	6	8	81.898	6	6
88.696	2	7	88.236	5	5
92.978	6	8	90.157	6	6

TABLE I—*Continued*

TITANIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
5503.885	6	6	5780.838	4	4
11.798	4	3	86.039	8	5
12.538	6	8	5804.388	3*n*	4
14.364	6	8	23.764	3	4
14.541	6	8	66.508	7	9
65.507	5	5	80.334	2	4
5644.180	5	7	99.338	6	6
48.638	5	4	5918.600	3	3
62.211	5	8	22.160	4	4
75.474	6	8	53.205	5	8
89.513	7	7	65.862	7	8
5702.727	6	4	78.582	6	8
11.944	6	4	99.003	4	4
15.168	5	4	6064.659	7	6
16.517	5	3	85.247	6	5
20.516	4	3	91.188	7	6
39.532	4	5	6126.239	7	7
40.035	4	3	6215.280	6	6
56.924	3	4	58.110	9	10
62.340	6	4	58.708	9	10
66.413	6	5	61.096	10	10
74.101	6	5			

TABLE II

MANGANESE

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
2812.845	3	3	2889.622	1	3
13.488	3	1	92.408	2	4
13.997	2	1	92.672	2	1
14.935	2	3	97.810	2	
15.618	2	1	98.006	2	
17.198	2	1	2900.565	1	4
17.986	3	1	02.212	2	
18.790	3		07.236	3	1
18.844	1		08.004	1	1
21.571	2	1	08.886	1	1
22.561	3	1	14.618	9	2
28.806	2	1	20.480	1	1
30.807	3	2	23.728	1	1
36.328	2	1	24.450	1	1
58.670	2		25.597	8	2
72.594	2	1	28.699	3	1
79.480	1	4	30.163	2	
82.916	1	1	33.068	7	8
86.688	1	4	34.024	3	1
89.550	2	6	35.663	2	1

TABLE II—*Continued*

MANGANESE

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
2939.314	7	8	3189.957	1	1
40.330	2	1	99.927	2	1
40.512	6	1	3201.121	2	1
41.047	3	1	02.539	2	1
41.694	1	1	03.736	2	1
41.755	1	1	06.894	3	1
44.410	1	1	12.862	5	3
49.224	8	9	16.928	3	2
53.031	1	1	23.228	2	1
63.615	2	1	24.745	4	2
78.582	2	1	26.015	3	2
3002.492	1	1	28.071	7	5
07.657	2	1	30.700	5	4
11.171	2	1	36.762	6	6
11.378	2	1	37.372	6	2
16.462	3	1	40.373	4	2
22.754	3	1	40.604	4	2
40.601	3	2	43.778	5	4
41.231	2	1	48.508	4	4
43.146	2	2	51.117	4	3
43.348	3	2	52.937	5	4
43.773	3	1	56.140	5	5
44.592	6	3	58.410	4	4
45.589	3	2	60.212	4	3
45.815	2	1	64.692	4	4
47.040	3	2	67.794	3	3
54.389	6	2	68.703	3	2
62.131	5	2	70.347	3	2
66.034	5	2	73.010	3	2
70.306	5	2	78.544	3	2
73.149	5	2	80.640	1	1
79.642	5	2	80.744	5	1
81.331	4	2	90.979	2	1
97.052	4	2	95.836	2	1
3110.686	3	2	96.027	2	1
13.037	1	1	96.872	3	2
13.808	2	1	98.220	3	3
15.473	3	2	3300.956	2	
20.342	2	1	03.277	3	2
22.886	1	1	04.898	2	2
25.023	1	1	07.008	3	1
26.858	1	1	08.785	3	1
32.206	2	1	11.906	3	2
32.805	2	1	13.199	3	2
36.960	2	1	13.514	3	2
42.674	3	1	14.419	2	1
48.188	5	2	14.898	3	2
58.731	1	1	16.328	3	2
59.946	2	1	16.452	1	1
61.053	5	2	17.304	4	2
77.053	1	1	20.693	4	2
78.495	6	3	30.666	4	1

TABLE II—*Continued*

MANGANESE

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3343.729	3	2	3693.668	4	4
45.354	3	1	96.588	4	3
51.665	2	1	3700.302	1	1
54.652	1		01.733	3	2
66.226	2	2	06.074	4	4
3407.982	1	1	18.926	4	3
19.801	1	1	31.925	3	3
33.570	3	2	46.613	3	2
38.978	3	3	50.758	2	1
41.998	5	10*nr*	56.631	2	1
50.614	2	2	63.376	2	2
60.330	5*r*	10*nr*	67.686	2	1
74.064	4	8	68.261	3	1
74.136	4	8	98.262	2	
82.924	4	10*nr*	99.256	2	2
88.678	4	10*nr*	3800.551	2	2
95.845	5	8	01.909	2	2
97.526	3	8	06.709	3	1
3524.548		8	06.860	7	8
31.830	4	8	09.119	2	2
32.002	5	8	09.591	5	4
32.128	5	8	10.687	2	2
47.790	5	10	16.738	2	3
48.025	4	8	23.510	4	5
48.187	4	8	23.882	4	4
52.737	2	1	24.772	3	2
69.485	5	8	32.435	3	2
69.796	8	8	33.836	6	5
70.061	4	6	34.361	6	6
74.085	1		36.508	4	1
77.865	7	6	38.329	4	2
79.656	2	1	39.776	4	4
80.120	1		41.721	4	5
83.692	2	1	43.979	3	4
86.536	5	5	52.403	3	2
89.980	1		53.487	3	3
95.109	5	4	56.663	3	3
3601.279	1	1	64.107	3	2
07.530	6	6	72.046	3	2
08.532	6	6	72.956	3	2
10.298	6	6	73.373	3	1
19.407	6	5	89.448	3	3
23.794	5	4	96.347	2	2
29.740	4	3	98.362	3	3
60.405	3	3	3904.318	1	2
69.839	1	1	04.960	2	1
70.518	3	1	11.130	3	1
76.950	3	2	11.424	2	2
80.147	1	1	16.628	2	1
82.091	2	2	18.312	3	2
84.866	1	1	21.766	3	1
91.815	1		22.682	3	1

TABLE II—*Continued*

MANGANESE

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3923.339	2	2	4070.265	3	3
24.070	3	2	75.251	2	1
26.470	4	3	79.204	7	4
29.252	3	2	79.430	7	4
29.654	3	2	82.922	8	6
36.757	2	2	83.620	8	6
42.974	2	1	89.932	3	1
52.844	2	2	92.348	2	1
75.883	2	2	95.040	1	
77.081	2	2	95.245	2	1
80.156	1	1	4102.970	2	1
80.883	1	1	03.460	1	1
82.170	1	1	03.550	3	1
82.572	2	2	05.370	3	1
82.903	1	1	07.880	2	1
84.177	2	1	10.917	6	2
85.238	3	2	13.260	3	1
87.097	3	2	14.408	2	1
87.469	3	2	22.250	2	1
88.677	1	1	22.770	2	1
91.602	1	1	23.296	1	1
92.488	2	1	23.548	1	1
4001.190	1	1	31.130	6	3
03.259	2	1	31.560	1	1
08.030	2	1	34.618	2	1
11.534	2	1	35.033	4	2
18.093	8	6	37.275	3	
20.083	3	1	41.060	3	2
26.226	3	2	47.530	3	1
28.601	2	1	48.794	3	2
31.797	$18n^r$	7	55.497	2	1
33.057	$18n^r$	5	57.003	3	1
34.477	$18n^r$	5	76.594	4	3
35.724	5	5	89.992	4	3
37.556	3	1	4201.780	3	2
41.343	12	6	11.750	3	2
45.208	4	3	20.613	3	1
48.734	11	5	35.125	6	4
49.012	2	2	35.306	6	4
49.445	2	1	39.723	6	3
51.726	2	1	57.653	5	3
52.461	2	2	61.298	3	1
55.205	2	2	65.920	5	3
55.545	10	4	78.682	2	1
57.948	4	2	81.097	5	3
58.917	5	4	84.084	5	2
59.375	4	2	4300.203	2	1
61.741	5	2	12.546	4	1
63.553	7	3	23.403	1	1
65.049	3	2	74.939	3	2
66.221	2	1	81.701	3	2
67.998	3	1	82.639	3	1

TABLE II—*Continued*

MANGANESE

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
4408.084	2	1	4826.901	2	1
11.875	3	2	38.240	2	1
14.880	6	4	44.312	3	1
19.774	3	2	54.613	1	1
36.051	2	1	54.805	1	1
36.351	5	4	62.052	1	
47.160	4*n*	4	4965.976	3	1
51.584	6	5	74.349	1	
52.535	2	1	85.777	2	1
53.015	4	3	5004.910	3	1
55.012	5	3	10.366	2	1
55.320	5	3	29.818	1	1
55.820	5	3	30.643	1	1
57.051	5	2	74.806	2	1
57.555	5	4	86.775	1	
58.268	5	4	5117.944	3	
60.389	4	2	49.255	1	
61.090	6	4	50.937	2	1
62.037	7*n*	6	96.603	5	3
64.680	6	4	97.228	2	
70.138	6	4	5255.330	6	3
72.800	6	4	60.774	3	1
79.400	3	2	98.033	2	1
90.071	5	3	5341.068	8	5
91.645	3	2	77.623	7	4
96.647	3	1	88.526	6	4
98.898	6	5	94.679	7	3
4502.218	6	5	99.494	6	4
03.872	3	2	5407.429	6	3
23.407	2	1	13.690	4	2
42.451	3	1	20.371	6	6
44.427	2	1	32.553	4	2
48.589	3	1	70.644	7	4
4605.378	5	3	81.401	6	2
26.552	5	3	5505.874	3	5
71.694	4	1	16.774	6	4
4701.150	3	1	37.753	4	3
09.708	7	4	51.991	3	3
27.476	7	4	73.016	1	2
39.004	6	3	73.688	2	2
54.046	10*n*^{r}	6	5738.287	6	2
61.521	6	4	80.168	6	2
62.375	8	6	5816.837	5	2
65.852	6	4	48.951	4	2
66.414	6	4	6013.480	10	10
83.454	10*n*^{r}	6	16.631	10	10
4823.521	10*n*^{r}	6	21.794	10	10
25.600	2	1			

TABLE III

VANADIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3126.224	9		3237.879	3	
30.281	9		39.045	3	
33.336	9		41.177	2	
34.944	4		41.988	3	
36.527	3		43.287	2	
41.500	2		49.574	3	
42.488	4		50.776	3	
45.354	4		51.889	3	
45.979	3		52.911	3	
47.275	2		54.768	4	
55.405	2		55.653	3	
62.568	2		59.542	3	
64.740	3		61.086	3	
68.136	3		62.075	3	
83.407	15		63.239	7	
83.984	15		65.904	3	
85.402	15		66.090	3	
87.700	7		67.690	13	
88.079	2		71.129	12	
88.503	5		71.636	2	
89.078	2		73.025	3	
90.674	8		76.130	15	
93.917	4		77.946	3	
96.503	3		79.845	4	
98.098	10		82.533	3	
3200.005	4		83.310	5	
01.227	2		84.364	3	
02.376	10		89.388	3	
04.199	3		91.680	3	
05.268	3		98.148	5	
05.575	6		98.738	4	
07.404	7		99.089	3	
08.351	4		3308.258	2	
10.104	3		09.184	4	
10.433	2		19.024	3	
12.396	7		20.147	3	
14.751	4		21.551	3	
15.380	3		21.691	3	
17.119	4		24.395	3	
18.872	2		27.996	2	
26.109	3		28.407	2	
27.125	2		29.858	8	
27.418	3		33.573	2	
29.612	3		56.358		
30.647	3		63.553	3	
31.952	3		65.558	8	
33.191	3		66.897	3	
33.550	2		71.126	3	
33.866	2		74.048	2	
34.519	3		76.063	4	
36.580	3		77.393	4	

TABLE III—*Continued*

VANADIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3377.627	4		3528.205	2	
83.758	4		29.729	4	
84.608	3		30.765	5	
85.949	3		33.666	2	
87.390	2		33.743	6	
90.388	3		34.739	2	
90.773	3		38.236	3	
96.528	2		40.535	2	
97.585	3		42.657	2	
97.845	3		43.495	4	
3400.401	6		45.188	4	
01.349	3		45.343	4	
02.572	6		48.922	2	
03.366	3		51.506	2	
04.970	2		53.263	4	
05.160	4		55.147	2	
06.851	4		55.739	2	
08.010	3		56.241	3	
09.101	3		56.800	5	
14.203	3		57.165	2	
17.076	3		60.578	3	
18.534	4		62.156	2	
23.874	3		63.395	2	
25.075	4		66.275	4	
42.017	3		68.939	3	
42.330	3		71.042	3	
45.819	3		71.657	3	
54.900	3		73.518	3	
56.930	3		74.776	2	
57.157	3		75.135	2	
63.415	2		77.877	2	
79.843	3		80.826	2	
82.188	2		83.702	2	
85.931	5		89.752	3	
89.477	4		92.014	3	
93.165	4		92.534	2	
96.942	4		93.332	3	
97.038	3		3600.040	2	
98.203	2		05.585	1	
99.837	2		16.740	1	
3500.825	2		35.482	2	
03.199	2		39.045	1	
04.425	5		41.108	1	
05.687	4		43.881	1	
06.847	3		44.736	2	
12.024	2		45.626	1	
17.289	8		47.359	1	
19.165	3		48.995	2	
20.022	4		52.453	1	
22.565	3		56.728	2	
24.714	4		63.599	3	
25.767	3		65.151	2	

TABLE III—*Continued*

VANADIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3667 737	3		3772.154	1	
71.210	3		74.111	1	
72.414	2		75.720	1	
73.411	3		76.160	1	
75.705	4		78.680	4	
76.693	3		79.650	1	
80.119	3		81.409	2	
81.287	1		82.553	1	
83.123	4		84.671	1	
86.268	3		87.553	2	
86.703	1		90.326	3	
88.073	6		90.484	2	
90.277	6		93.619	2	
92.225	6		94.955	4	
94.627	1		96.473	1	
95.347	4		99.900	4	
95.866	5		3803.474	5	
98.027	1		03.786	2	
99.484	1		03.914	2	
3703.585	7		06.805	1	
04.703	5		07.507	2	
05.044	5		08.523	4	
05.580	1		09.609	3	
06.045	1		13.494	5	
08.728	2		15.533	3	
15.467	3		17.854	1	
19.017	1		18.246	4	
22.004	1		19.978	4	
22.201	1		21.490	3	
27.350	2		22.014	2	
29.047	1		22.902	3	
32.756	2		23.229	3	
34.426	2		23.985	2	
37.999	1		28.193	2	
38.762	2		28.572	4	
41.513	2		29.627	2	
45.807	2		31.041	2	
47.138	1		32.433	2	
47.992	2		33.816	2	
50.875	2		35.183	2	
51.783	2		36.065	2	
52.868	2		36.524	3	
53.288	2		37.630	1	
55.714	1		37.858	2	
56.043	1		38.998	2	
58.555	1		39.388	2	
59.319	2		40.148	1	
60.803	1		40.436	3	
61.444	1		40.751	1	
63.148	2		41.901	2	
69.079	2		43.001	2	
70.541	1		43.496	1	

TABLE III—*Continued*

VANADIUM

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Arc	Spark		Arc	Spark
3844.446	2		3899.139	1	
44.896	2		3900.180	2	
45.018	2		01.244	2	
45.451	2		02.253	5	
52.408	2		03.260	2	
55.367	6		04.215	2	
55.839	7		06.746	3	
56.670	2		08.317	1	
58.691	2		10.791	2	
59.926	2		12.202	3	
60.637	2		12.883	2	
62.237	2		14.329	2	
62.497	1		16.415	1	
63.399	1		20.497	2	
63.877	1		21.906	3	
64.857	4		22.429	4	
67.642	3		24.659	4	
71.091	2		25.246	4	
72.747	1		27.932	3	
75.897	5		30.020	4	
76.096	4		31.347	3	
85.785	1		34.026	3	
86.590	2		35.143	2	
90.188	5		36.288	2	
91.859	3		38.216	2	
94.049	2		39.339	2	
96.159	2		42.008	3	
97.081	2		43.666	3	
98.281	2				

II. EFFECT OF CAPACITY AND SELF-INDUCTION ON THE WAVE-LENGTHS OF THE SPARK LINES

In view of the work of Exner and Haschek,[1] Haschek,[2] Kent,[3] Eder and Valenta,[4] Kayser,[5] Middlekauff,[6] Cooper,[7] and others on spark spectra, tests were made for the possible effect on the wave-lengths of the spark lines in consequence of varying the capacity and self-induction in the secondary circuit.

[1] *Sitz. der Kais.-Akad. der Wiss. in Wien*, 1897.

[2] *Astrophysical Journal*, **14**, 181, 1901.

[3] *Ibid.*, **17**, 286, 1903.

[4] *Ibid.*, **19**, 251, 1904.

[5] *Handbuch der Spectroscopie*, **2**, 297, 308–310.

[6] *Astrophysical Journal*, **21**, 116, 1905.

[7] *Ibid.*, **29**, 329, 1909.

APPARATUS AND METHODS

The apparatus used was the same as that already described, with the addition of self-induction in series in the spark circuit. This self-induction was produced by three coils of No. 10 copper wire, each 1 meter long and having 106, 206, and 186 turns respectively. The first and second were 8 cm in diameter and the third 15 cm. The second was mounted within the third and the whole was so arranged that the self-induction could be varied within the limits, 0.00007–0.0012 henry, without stopping the spark.

In determining the wave-lengths of the spark lines given in the tables, the capacity in the circuit was 0.016 micro-farad, but no self-induction was used. Throughout the spectra there was no evidence of a shift of the spark lines if the center of gravity of the lines is considered, but many lines become asymmetrical.

As several observers have announced detections of shift under varying conditions, rigorous methods were applied in the present work. In the beginning a test was made to see if any shift was produced by not having the arc and the spark in the same position. The arc was kept in its usual position, and, with the circuit conditions constant, the spark was displaced relatively. With the focusing lens fixed, plates were taken when the spark terminals were placed six or seven inches farther from the slit than the arc, when it was displaced perpendicularly to the line of the grating and the slit as far as the illumination of the grating permitted, and also when it was changed from place to place while the spark was running. No test gave the slightest evidence of a shift. In the last case a shift would have affected the definition of the lines, but they remained perfectly sharp.

Though these results showed that it was unnecessary to be especially cautious in regard to having the arc and the spark in exactly the same position, the precaution was taken in all of this work.

The titanium and the manganese sparks were subjected to varying conditions of capacity between the limits 0.0005–0.03 micro-farad, and of self-induction between the limits 0.00007–0.0012 henry. Special attention was given to those titanium lines in the region near λ 3900 which have been specially investigated by Kent. Not only were the spark spectra taken successively with the various combinations of capacity and self-induction, but also with variations of both

without stopping the spark or changing the plate. In the latter case a shift of the sharp lines would have caused blurring, which, however, was not observed. In order to detect mechanical shifts the method of half-time exposure was used.

GENERAL EFFECTS

The general effect of the increase of current from 10 amperes to 35 amperes was to intensify the lines and diminish the time of exposure. No reversals were found.

Self-induction eliminated the air lines, lengthened the time of exposure, caused the enhanced lines to revert to unenhanced lines, changed the characteristics of the spark spectra to those of the arc, and caused broad lines to become asymmetrical and nebulous, generally toward the blue end of the spectrum. Many lines sharp when only capacity was used showed the same effect when self-induction was introduced.

Increase of capacity shortened the time of exposure, broadened many lines, and made the enhanced lines asymmetrical and nebulous, generally toward the red end of the spectrum. Large capacity caused a few lines to reverse.

SHIFT OF LINES

The titanium lines, which, according to some observers, show a large displacement, are lines that are either enhanced or are broad in the arc spectrum, and this fact seems to be the cause of the disagreement among investigators as to whether a true shift occurs. Kayser points out the difficulties of determining a true shift in regard to broad and asymmetrical lines. A true shift should mean a shift of the center of gravity rather than the shift of the maximum point of intensity; but as there is at present no way of accurately determining the true center of gravity owing to the variations of the area of the lines, due to the time of exposure and of development, the center of maximum intensity is assumed to be the center of gravity.

So far as the writer has been able to discover, all calculations of shifts have been based on the position of the maximum intensity, which was found to vary slightly under different conditions of capacity and self-induction in the case of broad lines. Two of the titanium lines, λ 3900.53 and λ 3913.45, observed by Kent, are enhanced, and

the other two, λ 3904.77 and λ 3998.64, are broad and asymmetrical in the arc spectrum and also in the spark if self-induction is used. All are subject to asymmetrical broadening under changes of capacity and self-induction. Accordingly in the present work there arose the difficulty of determining not only the true center of gravity but also the real point of maximum intensity of these lines.

Since this work was done, Janicki[1] has investigated the shift of spectral lines, and he thinks that the apparent shift of the titanium lines found by Kent, and later by Kent and Avery,[2] is due to the asymmetry of the reversals.

CONCLUSION

The conclusion drawn from the evidence furnished by this investigation is that no determinable shift between arc and spark lines, or between spark lines subjected to different circuit conditions has been found.

My thanks are due Professor Ames, under whose supervision the work was conducted, Mr. L. E. Jewell, for his advice and assistance, and Dr. Anderson and Dr. Pfund, for their suggestions.

[1] *Annalen der Physik* (4), **29**, 866, 1909.

[2] *Astrophysical Journal*, **27**, 70, 1908.

THE SPECTRUM OF COMET 1908 *c* (MOREHOUSE)

By H. ROSENBERG

In the period between October 19 and November 16, 1908, I secured a series of spectrograms of comet 1908 *c* at the Observatory of the University of Göttingen. The ultra-violet prismatic camera by Zeiss was available for the purpose. It had previously served me for spectroscopic investigation of comet 1907 *d*,[1] and had been provided, in the meantime, with a new prism of very much smaller refracting angle, whereby the dispersion was reduced to about one-third of its former value. I had also attached to the prismatic camera a small and fast hand-camera, having a double anastigmat by Goerz, known as the "Celor," of aperture 25 mm, focal length 120 mm, and taking a plate 9×12 cm; in front of the objective of this a grating replica by Thorp was placed. The plates used were the "Agfa-Chromo" of the *Actien-Gesellschaft für Anilinfabrikation*, which have a high sensitiveness for the yellow-green region of the spectrum. Plates were developed with rodinal, 1:15, at a temperature of 15° C.

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Number	Instrument	Date 1908	Sidereal Time		Remarks
			Beginning	End	
683....	Prismatic camera	Oct. 19	21^h34^m	23^h04^m	Faint, particularly the tail
684....	Prismatic camera	Oct. 20	21 15	0 15	Very strong
698....	Prismatic camera	Oct. 29	23 32	1 02	Rather strong
719....	Prismatic camera	Nov. 11	21 15	22 45	Somewhat diffuse
721....	Prismatic camera	Nov. 16	21 45	23 15	Best plate
a.....	Goerz Objective	Oct. 19	21 35	23 05	
b.....	Goerz Objective	Oct. 20	21 16	0 16	
c.....	Goerz Objective	Nov. 11	21 16	22 46	
d.....	Goerz Objective	Nov. 16	21 46	23 16	Very weak

The following studies cover the general appearance of the spectrum of the comet, the establishment of as accurate wave-lengths as possible for the different monochromatic images, and the photometric determination of the distribution of intensity in the spectrum.

[1] *Astronomische Nachrichten*, **175**, 401, 1907.

GENERAL DESCRIPTION OF THE SPECTRUM

The plates obtained with the large prismatic camera were solely used for judging of the general appearance of the cometary spectrum, since the plates obtained with the Goerz objective, on account of their small scale, did not permit the detection of details.

The spectrum of comet 1908 *c* presents an entirely different appearance from that of comet 1907 *d*, which was similarly photographed by me with the same instrument. In that case, only the two characteristic bands at λ 4730 and λ 3880 were conspicuous, and the remaining knots could be recognized only with difficulty—perhaps a consequence of the strong continuous spectrum. The new comet exhibited five very strong monochromatic images at wave-lengths 390, 409, 428, 456, 468 $\mu\mu$, and a few other much weaker, but clearly recognizable, knots.

The presence or absence of a continuous spectrum could not be settled immediately from the mere appearance of the plates. The separate condensations lay so close together that it had to be left undecided, at first, whether the blackening of the intervening background was to be attributed to the action of these images, or to a continuous spectrum. I shall return to this point in discussing the photometric results. The remark should be made now that the comet was frequently investigated here, visually, for phenomena of polarization, but without permitting the determination of the presence of such phenomena with certainty. The reflected light, accordingly, probably constituted, in the case of comet 1908 *c*, only a very small fraction of the total light, at least as far as the visual spectral regions are concerned.

The images of the tail can be separately followed, in the case of the brighter condensations, for about 40′; at a greater distance from the head, they broaden and mingle together, so that they cannot be separately distinguished; but the tail is still recognizable at the edge of the plate, that is, at a distance of about 5° from the head. On plate No. 721, which is particularly sharp, the tails belonging to the five bright knots do not run out into the background in a faint and diffuse manner, but are bounded on both sides by a condensation of light, increasing as it goes outward, similar to the edges of a petroleum flame from a round burner. These edges appear so sharp that they

give the impression that we are dealing here, not with a band spectrum, but with purely monochromatic pictures of the comet in the particular wave-lengths.

I have not been able to detect on any plate a doubling of the single knots, such as was noted by several other observers, but with the slight dispersion and the relatively long focus of the instrument, the discovery of such delicate details would be, a priori, hardly expected.

DETERMINATION OF WAVE-LENGTHS

Wave-lengths are determined with a prismatic camera by assuming that one wave-length on the plate is known, and then by computing the remainder with a dispersion formula, or by graphic process. In the absence of a comparison spectrum, it is customary to obtain, before or after the exposure to the comet, the spectrum of a star with known lines, closely above and below that of the comet, and to reduce the unknown wave-lengths in the cometary spectrum, with the aid of the known star lines. This method can be regarded as justified only by necessity, for it is clear that every difference in pointing on the comet and star, every difference in flexure in the instrument in passing from star to comet, and every jar which might affect the position of the camera relative to the guiding telescope, enter with full weight into the result, and may thus falsify the results. The wave-lengths determined by different observers according to this method, therefore, are in some instances entirely out of agreement.

It is more desirable to obtain the wave-length of at least one of the knots in another manner. A slit-spectrograph with a known source of comparison light is commonly employed for this purpose, but it has the disadvantage of great faintness, unless the slit is opened rather wide. Inasmuch as no slit with much power was available, I placed the transparent replica of a Rowland grating in front of the above-mentioned double anastigmat of short focus and great light-power, placing the plane of the grating perpendicular to the optical axis of the instrument. With this arrangement a normal and somewhat weakened image of the object is projected in the center of the plate, and on both sides of it lie the spectra of the first order. It is now only necessary to expose on a bright star having sufficiently strong lines, either just before or after the comet exposure, and then the plate will furnish

all the data necessary for reduction, since any errors of the pointing or of flexure can be recognized by measuring the position of the normal images on the two plates, and can be allowed for in a reduction of the one spectrum to the other.[1]

With the relations of size in my arrangement, one Ångström unit corresponded to a distance of 0.008 mm, and the diameter of the separate condensations was about 0.05 mm. The instrument had enough light-power so that the three or four brightest knots in the cometary spectrum could be measured with satisfactory accuracy, with *α Lyrae* and *α Aquilae* as comparison stars. The following wave-lengths were obtained with the four brightest images from the four plates, *a*, *b*, *c*, and *d*.

Plate *a*	Plate *b*	Plate *c*	Plate *d*	Mean
3789	3806	3817	....	3804 Å. U.
3907	3898	3902	....	3902
4016	4931	4037	4036	4030
4275	4283	4272	4282	4278

On the basis of the mean values thus obtained, the plates of the prismatic camera, having much more detail, were reduced. After the Hartmann formula had been derived for each plate from the hydrogen lines $H\gamma$, $H\epsilon$, and $H\lambda$ in a star of the first type, the wave-lengths found above were identified for each plate with the corresponding condensations, and thence the wave-lengths were computed for the other knots, and a mean was found for each. Plates 684, 698, and 721 only were used for the reduction, as having sufficient sharpness for measurement, and plate 721 received double weight because it was by far the best defined. Settings were always made on the center of the knots only, as the edges were too diffuse, and, moreover, ran together with the adjacent images.

Two very faint condensations could be seen on the side of long wave-lengths, in addition to those visible on the measuring machine, but they were visible only when the plate was examined with the film toward a mirror or pressed upon white paper. A rough estimation

[1] The same result could be attained with a stereoscopic camera of large light-power, after placing in front of one of the objectives a direct-vision prism of high dispersion. It would be expected that the prismatic spectrum would have more intensity than the diffraction spectrum.

with a millimeter scale gave 0.51, 0.56 μ as the corresponding wave-lengths, so that we doubtless are dealing with the third and second carbon bands.

The following table gives the wave-lengths computed from the different plates for the centers of the condensations.

Plate 684	Plate 698	Plate 721	Mean	Remarks
3537	3544	3543	3542	?
3567	3582	3589	3582	4th Cyanogen (3590, 3586)
3700	3700	3700	3700	?
3808	3799	3805	3804	?
3893	3905	3897	3898	3d Cyanogen (3883, 3872)
4040	4026	4036	4035	?
4270	4285	4276	4277	?
4562	4564	4563	4563	1st Cyanogen (4606, 4578)
4684	4677	4683	4682	4th Carbon
5100	....	....	5100	3d Carbon
5600	....	....	5600	2d Carbon

These results appear to me to indicate that we can assert with certainty only the presence of the carbon bands in the spectrum of comet 1908 *c*. I should not venture to decide from my plates alone whether or not the identification of the computed wave-lengths of the cyanogen bands was correct, if these had not been confirmed by the results of Campbell with a slit-spectrograph; and this, particularly in view of the fact mentioned above, that the images of λ 3898 and λ 4563 appeared always sharply bounded and did not make the diffuse impression which the edges of a band would produce. The image at λ 3582, which very nearly merges with the one preceding it, is indeed somewhat diffuse. Of the second cyanogen band also I have not been able to find the faintest trace.

These results agree well, in general, with the measurements of Campbell and Albrecht,[1] Deslandres and Bernard,[2] Frost and Parkhurst,[3] and Hartmann,[4] although deviations of course occur in respect to details. In particular, the slit-spectrograph attached to the great Lick refractor permitted a much more complete resolution of the spectrum into separate lines. However, the wave-lengths found by

[1] *Lick Observatory Bulletin*, No. 145.

[2] *Comptes Rendus*, **147**, 774, 1908.

[3] *Astrophysical Journal*, **29**, 55, 1909.

[4] *Astronomische Nachrichten*, **181**, 21, 1909.

M. de la Baume Pluvinel and Baldet[1] cannot be well reconciled with my values, and the identification by Pickering[2] of the different images with the lines of the hydrogen series, I could not confirm.

SPECTRO-PHOTOMETRIC RESULTS

The remarkable intensity of the photographic rays in comparison to the visual brightness of comet 1908 *c*, made it appear desirable to compare the distribution of intensity in its spectrum with that of a star of known spectral type. The stellar spectra obtained simultaneously with that of the comet are wholly unsuited for such a comparison, however, as in guiding upon the comet they were usually so diffused that in measuring the degree of blackening it is not possible to obtain any clue to the wave-lengths corresponding to the different degrees of blackening. It therefore becomes a necessity to photograph, subsequently, stars suitable for such a comparison, as well as for deriving constants of the plate. In the arrangement of the observations, this purpose was not originally in mind, so that it did not become possible to derive all the data for the opacity curves from the plates themselves; for this investigation, I only had, on the same plates as the comet, the spectra originally taken for deriving the dispersion formula. Nevertheless, with the assistance of other plates, the relative distribution of intensity in the spectra of the comet and of the comparison star could be determined. In the discussion, plates 684, 719, and 721 were used, on all of which *α Lyrae* had been used as comparison star.

At first, both spectra were measured on all three plates under the Hartmann microphotometer, from 0.1 to 0.1 mm, the blackenings of *α Lyrae* were graphically represented and smoothed out by a curve bridging over the absorption lines. In order to determine the intensities corresponding to these smoothed-out blackenings the spectrum of *α Lyrae* was photographed with the prismatic camera on another plate of the same kind, with equal exposure time, once with full aperture, and the second time with the objective covered by a grating of which the absorption constant had been previously determined accurately. Both plates were similarly measured under

[1] *Comptes Rendus*, **147**, 666, 1908.

[2] *Astronomische Nachrichten*, **179**, 383, 1909.

the microphotometer, the blackenings laid down graphically and smoothed out as above, and then, on the basis of the known difference of brightness, the curve of blackening was computed for this plate, with the aid of which it was then easy to convert the blackenings into stellar magnitudes. The distribution of intensity thus found in the spectrum of *α Lyrae*, which was reduced to the zenith with data elsewhere obtained for the extinction at different wave-lengths, forms the basis for deriving the constants of the plates in question.

The curves of blackening derived in this way are collected in the following summary, in which the first column contains the measured blackening, and the remaining columns the corresponding intensities for the three plates, expressed in stellar magnitudes.

S	Mag. Pl. 684	Mag. Pl. 719	Mag. Pl. 721
38.0	(1.53)	....	....
40.0	2.37	(1.15)	(1.28)
42.0	2.74	2.23	1.77
44.0	3.00	2.72	2.33
46.0	3.24	3.01	2.73
48.0	3.47	3.22	2.98
50.0	3.67	3.42	3.16
52.0	3.86	3.63	3.32
54.0	4.08	3.86	3.46
56.0	4.39	4.14	3.63
58.0	(4.78)	(4.65)	3.83
60.0	....	....	4.06
62.0	....	....	4.32
64.0	....	....	(4.65)

The assumption cannot be rigorously accurate that the distribution of intensity in a particular spectrum as determined for one plate remains unaltered for other plates of the same kind; but it may be permissible, in view of the impossibility of obtaining the data necessary for reduction in a direct manner; and in our case, it has led to a satisfactory agreement in the results from the three plates. The values of extinction used are not definitive, and in their ultimate determination will require certain corrections, so that I give in the following tables, which contain the measured blackening for the three plates and the brightness derived therefrom, not only the values reduced to the zenith, but also the corrections applied for extinction. The separate columns contain, in order, (1) the position in the pris-

PLATE No. 684

mm	S_s	S_c	I_s	I_c	$I_c - I_s$	Δ_e
−3.35		37.8		(1.45)		
25		37.6		(1.33)		
15		37.7		(1.39)		
−3.05		37.3				
−2.95		39.0		2.08		
85	39.3	38.5	2.25	(1.87)	(−0.38)	−0.06
75	40.6	39.3	2.62	2.20	−0.42	6
65	41.7	40.2	2.82	2.48	−0.34	6
55	42.0	40.5	2.86	2.56	−0.30	6
45	41.7	39.3	2.82	2.19	−0.63	6
35	40.2	40.5	2.55	2.57	+0.02	6
25	39.1	39.8	2.19	2.38	+0.19	6
15	38.4	40.1	(1.89)	2.46	(+0.57)	6
−2.05	38.3	40.5	(1.84)	2.57	(+0.73)	7
−1.95	38.3	40.3	(1.84)	2.52	(+0.68)	7
85	38.4	40.6	(1.90)	2.59	(+0.69)	7
75	38.5	41.6	(1.95)	2.76	(+0.81)	7
65	39.0	41.1	2.18	2.69	+0.51	7
55	39.8	41.8	2.45	2.79	+0.34	7
45	41.5	42.6	2.83	2.90	+0.07	8
35	43.5	42.9	3.09	2.94	−0.15	8
25	45.6	44.7	3.37	3.17	−0.20	8
15	47.4	45.8	3.58	3.31	−0.27	8
−1.05	48.5	47.1	3.69	3.46	−0.23	8
−0.95	49.6	48.0	3.81	3.57	−0.24	9
85	50.4	48.8	3.89	3.65	−0.24	9
75	51.5	48.5	3.99	3.61	−0.38	9
65	52.4	48.6	4.09	3.64	−0.45	9
55	53.0	47.5	4.15	3.52	−0.63	9
45	53.5	47.0	4.21	3.46	−0.75	10
35	54.1	47.1	4.29	3.47	−0.82	10
25	54.6	47.6	4.37	3.54	−0.83	10
15	55.1	48.7	4.45	3.66	−0.79	10
−0.05	55.5	49.7	4.52	3.75	−0.77	11
+0.05	55.8	52.3	4.57	4.00	−0.57	11
15	56.2	55.1	4.65	4.35	−0.30	11
25	56.4	54.0	4.69	4.20	−0.49	11
35	56.5	53.6	4.72	4.15	−0.57	12
45	56.6	53.0	4.74	4.08	−0.66	12
55	56.7	52.1	4.76	4.00	−0.76	12
65	56.7	51.7	4.76	3.96	−0.80	12
75	56.6	51.9	4.76	3.98	−0.78	13
85	56.6	53.1	4.76	4.10	−0.66	13
+0.95	56.5	53.9	4.75	4.20	−0.55	13
+1.05	56.3	56.0	4.71	4.53	−0.18	13
15	56.1	56.9	4.69	4.70	+0.01	14
25	55.9	55.6	4.65	4.46	−0.19	14
35	55.6	55.6	4.61	4.47	−0.14	14
45	55.3	55.1	4.57	4.38	−0.19	15
55	55.1	57.2	4.53	(4.77)	(+0.24)	15
65	54.7	58.0	4.49	(4.94)	(+0.45)	15
75	54.1	58.0	4.40	(4.94)	(+0.54)	16
85	53.5	58.0	4.33	(4.94)	(+0.61)	16
+1.95	52.5	56.1	4.24	4.57	+0.33	17
+2.05	51.0	54.5	4.10	4.32	+0.22	−0.17

PLATE No. 684—*Continued*

mm	S_S	S_C	I_S	I_C	I_C-I_S	Δ_e
+2.15	49.6	52.9	3.98	4.12	+0.14	−0.18
25	48.1	51.8	3.84	4.02	+0.18	18
35	46.6	49.7	3.68	3.82	+0.14	19
45	45.0	48.0	3.51	3.66	+0.15	19
55	43.1	46.4	3.27	3.48	+0.21	20
65	42.4	46.8	3.19	3.54	+0.35	20
75	41.5	46.6	3.07	3.51	+0.44	21
85	41.0	45.9	3.00	3.43	+0.43	21
+2.95	40.6	46.4	2.93	3.50	+0.57	21
+3.05	40.3	44.0	2.87	3.21	+0.34	22
15	40.0	44.0	2.80	3.22	+0.42	22
25	39.8	44.2	2.75	3.24	+0.49	23
35	39.6	45.4	2.70	3.40	+0.70	23
45	39.4	45.4	2.64	3.41	+0.77	24
55	39.3	46.1	2.61	3.50	+0.89	24
65	39.2	45.5	2.60	3.43	+0.83	25
75	39.1	45.7	2.57	3.47	+0.90	25
85	38.6	45.3	2.35	3.43	+1.08	26
+3.95	38.4	43.9	(2.30)	3.26	(+0.96)	27
+4.05	38.2	42.2	(2.20)	3.05	(+0.85)	−0.28
15		41.7		2.99		
25		40.2		2.72		
35		39.4		2.48		
45		39.1		2.38		
55		38.7		2.22		
65		38.5		2.13		
75		38.2		(1.99)		
85		38.9		2.34		
+4.95		38.8		2.30		
+5.05		38.5		2.17		
15		37.6		(1.66)		
25		37.8		(1.80)		
35		37.4				
45		37.7				
55		37.0				
65		36.9				

NOTE.—The tabular data for plates 719 and 721, which are quite similar to the above, are omitted.—EDS.

matic spectrum measured in mm, reckoned from $H\gamma$ as the starting-point; (2) and (3) the smoothed-out blackenings in the spectrum of *α Lyrae* and the directly measured blackenings in the cometary spectrum; (4) and (5) the brightness of *α Lyrae* and of the comet expressed in stellar magnitudes reduced to the zenith; (6) the difference in brightness: comet minus *α Lyrae;* (7) the correction for extinction contained in this difference. It should be pointed out here, however, that the differences of brightness thus obtained are not absolute, but only relative: that is, that they are falsified for each plate by a con-

stant amount. From these values it is possible, therefore, to derive the color-gradation (*Farbtönung*) but not the surface-brightness of the comet. The values included in parentheses depend upon the extrapolation of the blackening derived for the very slight and very strong degrees of blackening.

The distribution of brightness in the spectrum of the comet and of *a Lyrae* is shown graphically in Fig. 1. The prismatic spectrum is retained for the abscissae, but the corresponding wave-lengths are indicated for every 50 $\mu\mu$. The Hartmann formula

$$\lambda = 1664.4 + \frac{[4.37720]}{n + 8.9045}$$

serves for a more accurate transformation of the prismatic spectrum.

An examination of Fig. 1 shows a very satisfactory agreement in the results of all three plates. It is true that the comet appears to be brighter in the ultra-violet on the last two plates than on the first—this is particularly true of the two knots at λ 358 and 354 $\mu\mu$—but I am still uncertain whether this appearance is real, as it is possibly due to a too rapid increase in the correction for extinction in the direction of the short wave-lengths. Apart from this slight difference, there is no marked alteration to be seen in the distribution of the intensity of the spectrum of the head on the three plates.

The monochromatic images of the comet, whose wave-lengths were determined above, are all indicated in the curves of brightness, but, at the same time, these permit us also to study more precisely the progression of the brightness of the background. We cannot decide with certainty, however, whether this background is due to the presence of a continuous spectrum, or to the overlapping of images of the head, which grow broader and fainter as they go outward, or to a sort of irradiation on the plate. We can only recognize this much, that the background must have a considerably bluer color than the comparison star; the distribution of intensity therefore corresponds by no means to that in the spectrum of the sun, which, as is well known, is redder than *a Lyrae*.

In order to exhibit these relations more clearly, a graphical representation is given in Fig. 2 of the differences: comet minus *a Lyrae*. The marked increase in the brightness of the comet in comparison with *a Lyrae* in the direction of shorter wave-lengths from λ 450 to

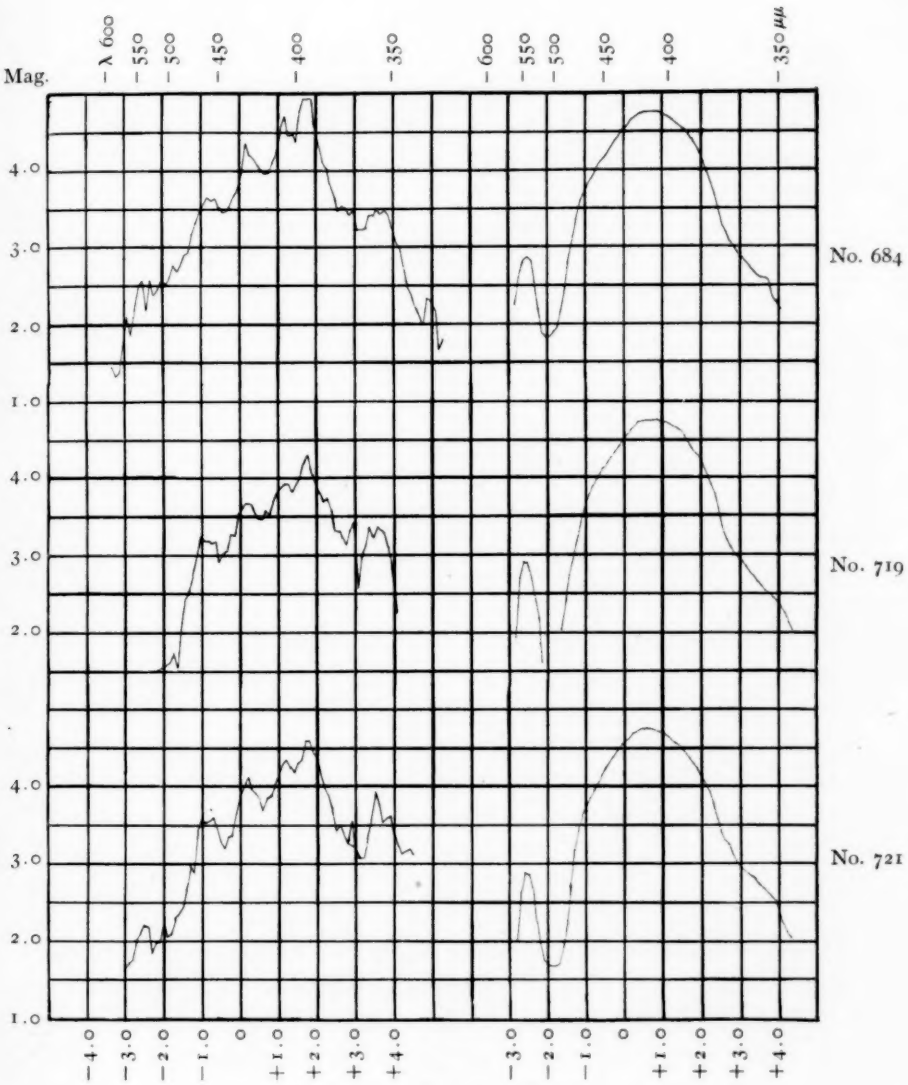


FIG. 1.—*I* Comet, *I* α *Lyrae*

350 $\mu\mu$ is very clearly shown. The strong maximum lying at about λ 510 and coinciding with the third carbon band conceals the background completely from λ 530 to 450, but, in view of the distinct gap at λ 540 on Plates 684 and 721, a further decrease in the relative brightness of the background toward longer wave-lengths might be inferred. I do not regard it as established that the carbon band is actually so strong as appears from the curve, inasmuch as the plate has a minimum sensitiveness at just this point, so that the blackenings of *a Lyrae*, which hardly exceed the threshold values, made necessary an extrapolation of the already very flat curve of blackening.

Without wishing to commit myself to the view that the blackening of the background is certainly a consequence of the continuous spectrum, I believe that it might, nevertheless, be of interest to discuss the matter from this point of view. As above remarked, the distribution of the brightness is entirely different from that in the solar spectrum. The increase in intensity toward shorter wave-lengths is stronger than is to be found even in the hottest white stars, and the activity of the photographic rays in contrast to the visual rays is fully confirmed by the blue color of the comet. Is it not, therefore, natural to think that the sunlight is reflected by the smallest particles of the cometary material in a similar manner as in our atmosphere, and then to compare the color of the comet with the blue color of the diffusely reflected sky-light? This assumption is not contradicted by the fact that visual observations did not indicate polarization, for a glance at Fig. 2 shows that the brightness of reflected light is excessively small in comparison to that of the intrinsic light of the comet just in the spectral region most effectual visually. Conditions are different for the photographically active rays; here, under circumstances, a distinct effect of polarization ought to be noticeable in case this hypothesis of a diffuse reflection corresponds to the facts; but no observations on this point in regard to comet 1908 *c* are known to me.

Under the assumption that the comet reflects the sunlight in the manner here indicated, we may regard the portion producing the continuous spectrum as a cosmical dust-cloud, in which we may first regard the diameter of the separate particles to be something of the order of magnitude of the wave-lengths, and it will be possible to reach an approximate determination of the mass of the comet, if we

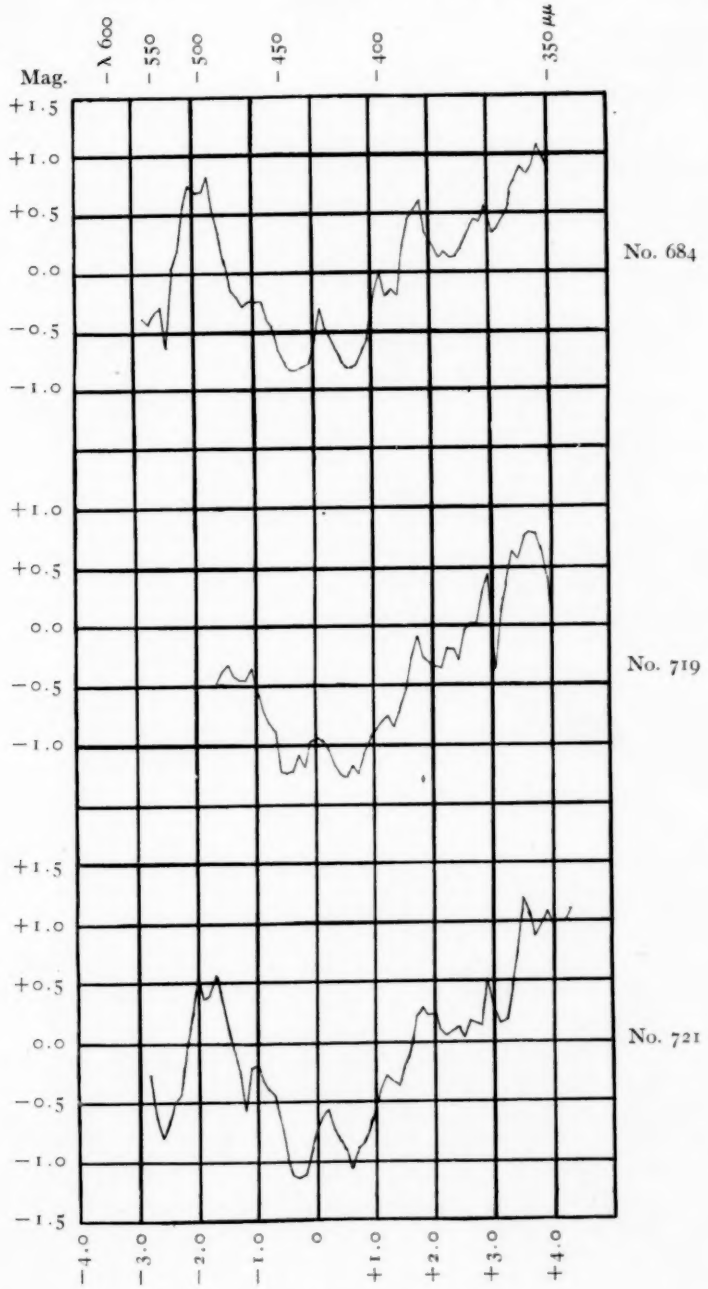


FIG. 2.—$I_c - I_s$: Comet — α *Lyrae*

succeed in determining the ratio of the surface brightness of the continuous spectrum of the comet to that of the sun.

On all three plates *α Lyrae* was photographed as a comparison star with the telescope at rest, i. e., the daily motion was used for broadening the stellar spectrum so that, with the focal length of the instrument (110 cm) and with the known declination of the star, the exposure-time could be computed corresponding to a definite width of spectrum. From this we may derive the quantity of light radiated for any wave-length from a star upon the unit of surface (1 sq. mm), and compare it with the corresponding surface-brightness in the comet's spectrum. The conversion will, of course, not be rigorously exact, since the relation between intensity, exposure-time, and blackening is not accurately enough known for such large differences; nevertheless, we may obtain an idea of the order of magnitude with which we have to deal. I based[1] my computation upon the formula $S=it^p$, where the exponent p was found to be 0.838, from other exposures with the kind of plates used.

For equal blackening in the spectrum of star and comet we therefore obtained the ratio of surface brightness: $\frac{i}{i'}=\left(\frac{t'}{t}\right)^p$ where t indicates the exposure-time for the comet, and t' that for the star for a width of one element of surface (1 mm). In order to obtain the difference of brightness, comet minus star, for any given wave-length, it is necessary to add the quantity I_c-I_s, to be taken from Table II, for the corresponding wave-length, to the value thus computed, and convert it into magnitudes. I have computed this value for the wave-length 420 $\mu\mu$, which falls in closely with the maximum of sensitiveness of the plate, and I find this result: If the star's light of this wave-length is spread over one square mm of the plate, then the surface brightness thus produced by *α Lyrae* is, for the three plates, brighter by, respectively, 6.70, 6.54, 6.34 stellar magnitudes than the corresponding brightness produced by the comet in the same time. The measures of brightness on the comet here refer only to the central zone of greatest opacity having a width of only about 0.3 mm.

The sun is visually about 26.8 magnitudes brighter than *α Lyrae*.

[1] K. Schwarzschild, "Beiträge zur photographischen Photometrie der Gestirne," *Publicationen der von Kuffner'schen Sternwarte*, **5**, 1900.

This value for λ 420 $\mu\mu$ is diminished by about 1.2 magnitudes when we consider its distinctly more red color, and since the sun at mean distance would cover a surface of 82.28 square mm, with the focal length of this instrument, the surface-brightness produced by it would be 20.8 magnitudes brighter than that produced by *a Lyrae* on a unit of surface. Hence, we derive the difference of the surface-brightness of the sun and the comet for λ 420 $\mu\mu$ to be 27.5, 27.3, and 27.1, for the three plates.

If we now imagine the head of the comet to be replaced by a surface of albedo 1 reflecting diffusely according to the Lommel-Seeliger law, we obtain (if, for simplicity, we place the angle of incidence equal to the angle of emergence, and equal to half of the angle of phase) for the ratio of its surface-brightness to the brightness of the solar surface the following values converted into magnitudes: October 20, mag.=14.3; November 11, 13.9 mag.; November 16, 13.9 mag.

The actual surface-brightness is about 200,000 times less, which shows that we are dealing with matter so thinly scattered that there can be no talk of occulting and shadow-casting. This result is in good agreement with the fact that photometric observations of stars which were covered by the comet do not indicate any measurable diminution of the stars' light.

If we now call the radius of the comet's head R, and let ρ be the radius of the separate particles, which we will first assume to be 0.1 μ, and the number of all the particles present in the comet's head be N, and the ratio of their computed maximum intensity to the observed brightness be c, then we may place $\frac{\pi R^2}{c}=N\pi\rho^2$. Let us designate, by P the space occupied by all the particles together, the space occupied by the comet's head by P_1, then we shall get a measure of density of the comet from the quotient $\frac{P}{P_1}=N\left(\frac{\rho}{R}\right)^3$, which, furthermore, varies with the first power of ρ.

The diameter of the comet's head comes out from the measures on plates 684 and 721 as, in the mean, 5.8 times the earth's diameter, whence we compute, for albedo=1,

$$N=6.6\times10^{23}, \quad \frac{P}{P_1}=1.3\times10^{-20}.$$

If we take the volume of the earth as the unit for P_1, then $P=3.2 \times 10^{-18}$, and the mass of the comet, for a specific gravity $=1$, expressed in terms of the earth's mass, will be

$$M=5.8\times 10^{-19}.$$

For albedo 0.1, which would correspond to a relatively dark surface, all these values would have to be multiplied by 10.

The observed surface-brightness of the comet permits us, also, to derive a maximum brightness for ρ. We shall assume for that, that a cosmical dust-cloud would still just appear to us as a continuous surface if at least one particle of matter occurred on every square second of surface. For the closest approach of comet 1908 *c*, $\log \triangle = 0.015$, at which distance 1″ corresponds to a length of about 750 km. Since we have found the surface of the particles for albedo $=1$ to be about 200,000 times less than that of a continuous surface, we thus derive for the maximum value

$$\rho \text{ max.} = \sqrt{\frac{750^2}{200{,}000} \cdot \frac{1}{\pi}} = 0.95 \text{ km};$$

and, at the same time, for the maximum value of the mass of the comet's head 5.5×10^{-9} times the earth's mass. For albedo 0.1, these values would increase, respectively, to 3.0 km and 17.4×10^{-9}.

In addition to the spectrum of the head, I also made, on plate 721, photometric measurements on the caudal images corresponding to the five bright condensations at λ 3898, 4035, 4277, 4563, and 4682, out to a distance of 1.6 mm from the head at intervals of 0.2 mm apart, in order to determine the decrease in brightness at the corresponding wave-lengths. The following table gives the distance of the head in mm, the corresponding blackening, as well as the brightness derived therefrom by means of the curve of blackening after it had been diminished by the intensity of the image of the head itself, so that the values of brightness expressed in magnitudes represent the falling-off in light for all the images of the tail in a manner directly comparable. Beginning with those of shortest wave-length, which are denoted by 1 to 5, they are:

mm	1, λ 3898		2, λ 4035		3, λ 4277		4, λ 4563		5, λ 4682	
	s	mag.	s	mag	s	mag.	s	mag.	s	mag.
0.0	55.7	0.00	53.4	0.00	53.0	0.00	49.2	0.00	47.9	0.00
.2	52.0	0.28	52.7	0.05	50.8	0.13	46.9	0.25	45.6	0.31
.4	50.8	0.37	50.6	0.21	48.8	0.33	46.4	0.31	43.9	0.67
.6	48.3	0.58	49.2	0.32	47.9	0.42	44.7	0.61	43.9	0.67
.8	47.2	0.71	48.7	0.37	47.4	0.48	44.0	0.77	43.7	0.73
1.0	47.0	0.72	47.4	0.51	46.5	0.59	43.5	0.93	43.3	0.86
.2	46.1	0.86	47.3	0.52	46.2	0.63	43.2	1.02	43.2	0.89
.4	44.8	1.09	45.9	0.71	45.4	0.76	43.2	1.02	42.8	1.00
.6	44.5	1.15	46.0	0.69	44.6	0.92	43.0	1.06	42.4	1.10

The progression of these figures, of course, indicates clearly that the falling-off in the brightness in the caudal images of different wave-lengths is by no means the same, but that it is much more steep for images 1, 4, and 5 (for which it runs almost alike) than for images 2 and 3, which again are in good agreement with each other. Knots 1 and 4 were identified with the cyanogen bands, 5 with the fourth carbon band, while the element responsible for images 2 and 3 is thus far unknown. It is not possible to decide from this meager material whether we may be allowed to infer a closer relationship of the kinds of light in question from their photometric behavior; but it is of interest to note that the unidentified lines at λ 4035 and 4277 were also observed in the spectrum of comet 1907 *d*.

Even if the results of this investigation are quite unsatisfactory, nevertheless they show that with a correct arrangement of observations many valuable hints may be expected from this procedure, and in future apparitions of comets it will not be permissible to neglect the photometric evaluation of the plates. For establishing the constants, it is sufficient to photograph a bright star, with measurably altered aperture of the objective and equal exposure-time on the same plate as the comet; or to introduce a change in the exposure time for constant objective aperture as a factor of measurement. It also seems to me to be important that both visual and photographic observations should be made for investigating the phenomena of polarization exhibited by comets.

GÖTTINGEN
June, 1909

ON THE ABSORPTION OF LIGHT IN SPACE[1]

SECOND PAPER

By J. C. KAPTEYN[2]

1. *Data on which the investigation is based.*—After deriving the results of my first paper,[3] which make it probable that there is an appreciable amount of selective absorption of light in space, I was naturally anxious to obtain some quantitative determination of this loss. In the present paper I have tried to carry through such a determination.

The phenomenon must manifest itself in this, that, *ceteris paribus*, the more distant stars will be redder than the nearer ones. As a measure of average distance we may take the apparent magnitude and proper motion. The tables of *Publications of the Groningen Laboratory*, No. 8, furnish values which, in the mean of many stars, must give a fair approximation. As a measure of the degree of redness we may take the difference between the photographic and visual magnitudes. For both of these magnitudes excellent data are furnished by the Harvard observations. I have used the values of the difference: Mag. *Draper Catalogue* − Mag. *Harvard Revision*,[4] which I will indicate by:

$$Phot. - Vis. \qquad (1)$$

In order to operate with stars of which the spectral class is well known, I have used exclusively stars contained in the classifications of Miss Maury,[5] and of Miss Cannon for the *northern* stars.[6] I have in general restricted myself to those stars contained in these two works for which reliable proper motions can be found in the best

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 42.

[2] Research Associate of the Mount Wilson Solar Observatory.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 31; *Astrophysical Journal*, **29**, 46, 1909.

[4] *Harvard Annals*, **50**.

[5] *Ibid.*, **28**.

[6] *Ibid.*, **56**. The *southern* stars of Miss Cannon were not used because the difference (1) is unknown for them.

sources. But where some extension seemed particularly desirable, I have somewhat increased the number by deriving proper motions myself from all the catalogues accessible to me. In this way I finally obtained data both for distance and for degree of redness for 1433 stars, of which 441 are contained in the work of Miss Maury, and 992 in the work of Miss Cannon.

The amount of change d in the difference *Phot.* − *Vis.* for unit distance has been determined for each sufficiently numerous group of stars belonging to one and the same spectral class.

2. *The unknown quantities a, b, c, d.*—Meanwhile, the quantity *Phot.* − *Vis.* is a very defective measure of redness, for it depends on the definition adopted for photographic magnitude. Whether for stars of different apparent magnitudes a definite amount of *Phot.* − *Vis.* will correspond to the same amount of real redness depends on this definition. Owing to the imperfect way in which, up to the present time, photographic magnitude could be defined, we must expect that, even where we have to do with stars at the same distance, the difference of photographic and visual magnitude will vary with the apparent magnitude. For the values of *Phot.* − *Vis.* corresponding to the Harvard data this variation is very pronounced.

Furthermore, as I pointed out in my first paper, an increase of *Phot.* − *Vis.* with distance, even if we have to do with stars of equal apparent magnitude, does not necessarily mean space-absorption. For, the apparent magnitude being the same, the more distant sta s will have greater *luminosity* (i. e., total light power); and we cannot be sure a priori that, even with stars which, on account of the similarity of their spectral lines, have been placed in the same class, the degree of redness does not vary with the luminosity.

Let

π = parallax;

m = apparent magnitude;

M = absolute magnitude = apparent magnitude at unit of distance ($\pi = 0''.1$)

$$= m + 5 + 5 \log \pi; \qquad (2)$$

$$\Delta = \text{distance} = \frac{0''.1}{\pi}; \qquad (3)$$

An objection may be raised against both formulae (2) and (3). In the absence of data for the individual values of M and Δ in equation (4) we may expect good results if we introduce values which are correct on the average. Now, however,

and

a, b, c, d constants to be determined by observation.

For the reasons given I have put

$$Phot.-Vis.=a+bm+cM+d\Delta \qquad (4)$$

where

b=change in redness for a change of one unit in the *apparent* magnitude;
c=change in redness for a change of one unit in the *absolute* magnitude;
d=increase in redness per unit of distance; that is, 32.6 light-years.

3. *Determination of b.*—I will begin with the determination of b. In the subsequent derivation of c and of d, the main unknowns of the problem, the value of b is almost entirely eliminated. A somewhat rough determination of b would therefore have been sufficient for our main purpose. If, notwithstanding this, I determine b as accurately as possible from the present data, it is in order to bring out a fact that seems well worth study for its own sake.

To begin with, I have subdivided the stars of each spectral class into 3 or 4 groups according to the amount of their proper motion. Besides this, I have kept the stars near or in the Milky Way (between galactic latitudes $\pm 20°$) separate from the rest, at least in most cases. For each of the subdivisions thus obtained, in which the data promised even a moderately good determination, I have computed the value of b. The following results were obtained. The groups within galactic latitude $\pm 20°$ have been marked with a g, the extra-galactic groups, with an e.

TABLE I
CANNON STARS

Spectral Class	Position	P. M.	b	Number of Stars	Weight
B3	g	$\leqq 0''.020$	$b=-0.124-0.74c-2.1d$	7	13
	g	$0''.021-0.029$	$=-.193-0.83c-1.1d$	6	15
	g	$\geqq 0.030$	$=-.181-0.97c-0.0d$	8	24
B5	e	$\geqq 0.025$	$=-.241-0.37c-2.7d$	6	7
	g	$\leqq 0.020$	$=-.171-1.09c+0.7d$	9	13
B8	e	$0.021-0.029$	$=-.238-0.48c-2.5d$	6	6[5]

if π be the average parallax of a certain class of stars, then Δ, computed by (3), will *not* generally be the average distance, nor will M computed by (2) be the average absolute magnitude. Owing to the relative smallness of the deviations of the real parallaxes from those computed by our table (cf. *Gron. Publ.*, No. 8, pp. 20–23) this objection cannot be considered as very serious, however.

TABLE I—*Continued*

Spectral Class	Position	P. M.	b	Number of Stars	Weight
	e	$\geqq 0.030$	$= - .290 - 0.19c - 2.9d$	9	65
A	e	0.026–0.069	$= - .269 - 0.50c - 1.7d$	22	29
	e	$\geqq 0.070$	$= - .217 - 0.12c - 1.8d$	14	9
	g	$\leqq 0.025$	$= - .187 - 0.72c + 0.9d$	6	22
	g	0.026–0.054	$= - .287 - 0.81c - 1.2d$	13	19
A2	e	0.026–0.049	$= - .212 - 0.14c - 0.7d$	8	15
	e	0.050–0.089	$= - .306 - 0.60c - 1.0d$	10	11
	e	$\geqq 0.090$	$= - .167 - 0.17c - 0.7d$	8	11
	g	$\leqq 0.025$	$= - .305 - 0.23c - 6.9d$	6	18
A3	e	0.031–0.079	$= - .108 - 0.68c - 1.1d$	7	10
A5	e	0.041–0.109	$= - .367 - 0.50c - 1.2d$	11	14
	e	$\geqq 0.110$	$= - .056 - 0.73c - 0.3d$	8	18
	g	$\geqq 0.080$	$= - .093 - 0.44c - 1.3d$	7	11
F5	all	$\geqq 0.200$	$= - .009 - 0.42c - 0.3d$	7	8
G	all	$\geqq 0.200$	$= - .409 - 1.29c - 0.7d$	9	18
G5	e	$\geqq 0.130$	$= - .294 + 0.79c - 1.1d$	7	12
K	e	$\leqq 0.035$	$= - .380 - 0.69c - 2.1d$	14	36
	e	0.036–0.069	$= - .151 - 0.68c - 0.9d$	11	36
	e	0.070–0.139	$= - .324 - 0.69c - 0.5d$	11	30
	e	$\geqq 0.140$	$= - .281 - 0.44c - 0.4d$	13	50
	g	$\leqq 0.035$	$= - .495 - 0.69c - 1.4d$	8	20
	g	0.036–0.099	$= - .370 - 0.92c - 0.4d$	7	25
	g	$\geqq 0.100$	$= - .506 - 0.52c - 0.8c^2$	4	18

TABLE II

MAURY STARS

Spectral Class	Position	P. M.	b	Number of Stars	Weight
IV	all	$\leqq 0.''030$	$b = -0.1755 - 0.63c - 3.0d$	9	25
V	all	$\leqq 0.033$	$= - .092 - 0.64c - 2.0d$	6	18
	all	$\geqq 0.034$	$= - .228 - 0.79c - 1.3d$	7	10
VII	e	$\geqq 0.041$	$= - .038 + 0.06c - 2.1d$	4	12
	g	$\leqq 0.035$	$= - .139 - 0.96c - 0.2d$	4	21
	g	$\geqq 0.036$	$= - .129 - 0.45c - 1.7d$	5	20
VIII	e	$\leqq 0.040$	$= - .044 - 0.80c - 1.2d$	6	8
	g	$\geqq 0.044$	$= - .180 - 0.46c - 1.2d$	5	9
IX	e	$\geqq 0.085$	$= - .154 - 0.27c - 1.4d$	7	13
	g	$\geqq 0.041$	$= - .093 - 0.43c - 1.7d$	5	5
XII	e	$\geqq 0.087$	$= - .228 - 0.84c - 0.6d$	9	16
	g	$\geqq 0.100$	$= - .244 - 0.99c - 2.0d$	8	18
XIV	all	$\leqq 0.030$	$= - .343 - 0.43c - 3.3d$	8	27
	all	$\geqq 0.040$	$= - .402 + 0.11c - 1.5d$	6	20
XV	e	$\leqq 0.055$	$= - .383 - 0.27c - 3.6d$	7	21
	e	$0.''056$–0.149	$= - .341 - 0.91c - 0.2d$	10	59
	e	$\geqq 0.150$	$= - .341 - 1.15c + 0.2d$	10	37
	g	$\leqq 0.050$	$= - .412 - 0.10c - 5.1d$	7	26
XVI	all	$\leqq 0.040$	$= - .345 - 0.97c - 0.5d$	4	10
XVIII	all	$\geqq 0.045$	$= - .335 + 0.50c - 2.7d$	8	19

I first combined these values into the following averages:

Spectrum	b	Weight	Prob. Error	
	Cannon Stars			
B3, B5, B8.....	$b=-0.190-0.78c-1.0d$	85	±0.037	(5)
A, A2, A3, A5.....	$=-0.230-0.50c-1.4d$	187	.025	(5)
F5, G, G5.....	$=-0.288-0.45c-0.7d$	38	.055	(5)
K.....	$=-0.331-0.64c-0.9d$	215	±0.023	(5)
	Maury Stars			
IV–IX(B3–A2).....	$b=-0.133-0.61c-1.7d$	141	±0.014	(6)
XII, *XIV*(=F5, G).....	$=-0.313-0.50c-2.0d$	81	.019	(6)
XV–XVIII(K–Ma).....	$=-0.356-0.61c-1.3d$	172	±0.013	(6)

The surprising result of this computation is, that *the value of b changes with the spectrum.* The numerical value of b increases with increasing redness of the stars. The fact is clearly shown and the results are fairly accordant for the two observers. As the coefficients of c and d in (5) and (6) are practically the same for the different spectral classes, this difference in the values of b is evidently independent of both distance and absolute magnitude.

If the value of b had been found the same for all the spectral classes, the fact that its value was not *zero* would not have been very surprising, for it would have meant simply that the range of one magnitude is different for the visual magnitudes of the *Harvard Revision* and the photographic magnitudes of the *Draper Catalogue.*

Now that we find *b different* for the different spectral classes, the difference of scale does not explain the fact, and we are compelled to conclude that even *for stars of one and the same spectral class*, the ratio

$$\frac{\text{Brightness in the violet}}{\text{Brightness in visual spectrum}} \tag{7}$$

for which we may probably put

$$\frac{\text{Brightness in the violet}}{\text{Brightness in the yellow}} \tag{8}$$

changes largely with the apparent magnitude. We cannot, however, maintain that it must necessarily be so for all spectral classes, because we do not know the exact relation of the photometric scales of the *Harvard Revision* and *Draper Catalogue.* But we may for instance maintain that if (7) is constant for the First Type stars, it is *not* constant for the Second Type stars.

Now, we cannot imagine this change in the ratio (7), or (8), with

the *apparent* magnitude to be a *cosmical* fact. If it were, the change would be dependent on the *absolute* magnitude, for it seems absurd to suppose that the ratio in question would change (after allowance for space-absorption) for one and the same star by a mere change in distance. Farther on, however, we will find that the value of c, which is a measure of the change of the ratio (7) or (8) with the *absolute* magnitude, is, for the present purpose, completely vanishing. Therefore the phenomenon under discussion must certainly be dependent on peculiarities in the visual, or, more probably, the photographic determination of magnitude.[1]

It seems well worth while to make out what these peculiarities are. For our present purpose, however, where, as already mentioned, the value of b plays a very unimportant part, the result of such inquiry will not be needed.

It will even be quite allowable to introduce provisional values of c and d into (5) and (6). As such I have adopted:

$$c=0.000 \qquad d=+0.0060$$

with which finally

Cannon Stars		Maury Stars		
B3 to B9	$b=-0.196$	IV–IX(B3 to A2)	$b=-0.143$	(9)
A to A5	$=-0.238$	XII–*XIV*(F5 to G5)	$=-0.325$	
F5 to G5	$=-0.292$	XV–XVIII(K to Ma)	$=-0.364$	
K	$=-0.336$			

4. *Equations of condition for the remaining unknowns.*—For the determinations of the other unknown quantities I have grouped the stars in the following way: I first divided the stars of each class of spectrum in two groups—the galactic stars (again marked g in all the following tables) between -20 and $+20$ degrees of galactic latitude, and the extra-galactic stars (marked e), having higher latitudes. These groups were then subdivided in 2 to 4 sub-groups, according to the magnitude of the proper motion. Taking averages in each of these sub-groups and for these averages computing the quantities

[1] A sort of Purkinje phenomenon for photographic plates is well known. Cf. Sheppard and Mees, *Investigations on the Theory of the Photographic Process* (London: Longmans, Green & Company, 1907), pp. 297 ff. Investigations are now in progress which doubtless will throw further light on the significance of this phenomenon for the problem in question.

(2) and (3), I obtained series of equations of condition like the following (μ representing total proper motion):

CANNON STARS

B3, *e*, $\mu \leqq 0''.020$ ..... $a+5.10b-2.2c+28.6d=-0.020$ (9 stars)
$0''.021$ to $0''.029$ ..... $a+5.16b-0.4c+13.2d=-0.190$ (4 stars)
$\geqq 0''.030$ ..... $a+5.01b+0.0c+10.4d=-0.038$ (8 stars)

As the coefficients of b in these equations are very nearly equal, I have reduced them to perfect equality by means of the values (9). For the stars of Miss Cannon, they were reduced to 5.00; for those of Miss Maury, which on an average are half a magnitude brighter, to 4.50. The following equations of condition were thus finally obtained:

TABLE III

CANNON STARS

Spectrum and Position	Limits of P. M.	Average P. M.	Equations of Condition			No. of Stars
B3 *e*	$\leqq 0''.020$	$0''.009$	$a+5.00b-2.2c+28.6d=$	0.000 resp.	$+0.002$	9
	.021 to 0.029	.026	$a+5.00b-0.4c+13.2d=$	$-.158$	$-.152$	4
	>0.030	.036	$a+5.00b+0.0c+10.4d=$	$-.036$	$-.042$	8
B3 *g*	$\leqq 0.020$	.011	$a+5.00b-1.7c+23.8d=$	$-.012$	$-.009$	37
	.021 to 0.029	.024	$a+5.00b-0.8c+13.7d=$	$-.003$	$-.003$	13
	$\geqq 0.030$	.041	$a+5.00b+0.1c+9.3d=$	$+.002$	$+.013$	14
B5 *e*	$\leqq 0.020$	.013	$a+5.00b-1.7c+21.3d=$	$-.049^5$	$-.040^5$	8
	.021 to 0.024	.021	$a+5.00b-1.2c+14.5d=$	$-.052$	$-.049$	1
	$\geqq 0.025$	.036	$a+5.00b+0.0c+10.2d=$	$-.070^5$	$-.060^5$	8
B5 *g*	$\leqq 0.020$	.013	$a+5.00b-1.7c+21.3d=$	$+.037^5$	$+.045^5$	12
	.021 to 0.024	.022	$a+5.00b-1.3c+13.7d=$	$-.050$	$-.046$	1
	$\geqq 0.025$	.035	$a+5.00b+0.1c+10.6d=$	$+.053$	$+.043$	10
B8 *e*	$\leqq 0.020$	.014	$a+5.00b-1.3c+20.8d=$	$-.021^5$	$+.000^5$	13
	.021 to 0.029	.023	$a+5.00b-0.7c+14.3d=$	$-.052$	$-.039$	8
	$\geqq 0.030$	.052	$a+5.00b+0.7c+8.1d=$	$-.049$	$-.079$	13
B8 *g*	$\leqq 0.020$	.012	$a+5.00b-1.6c+23.3d=$	$-.045$	$-.024$	11
	.021 to 0.029	.024	$a+5.00b-0.2c+14.7d=$	$+.053$	$+.065$	6
	$\geqq 0.030$	.047	$a+5.00b+0.3c+8.3d=$	$+.113$	$+.080$	9
B9 *e*	$\leqq 0.027$	.018	$a+5.00b-1.0c+17.2d=$	$-.044$	$-.013$	6
	.028 to 0.038	.029	$a+5.00b+0.0c+12.5d=$	$-.021$	$+.001$	3
	$\geqq 0.039$	.058	$a+5.00b+0.8c+7.4d=$	$-.028$	$-.071$	6
B9 *g*	$\leqq 0.020$	.012	$a+5.00b-1.5c+23.3d=$	$-.041$	$-.017$	7
	.021 to 0.029	.023	$a+5.00b-0.6c+14.5d=$	$-.054$	$-.043$	3
	$\geqq 0.030$	.047	$a+5.00b+0.5c+8.6d=$	$-.026^5$	$-.078$	4
A *e*	$\leqq 0.025$	.017	$a+5.00b-0.9c+18.5d=$	$+.032$	$+.086$	20
	.026 to 0.069	.045	$a+5.00b+0.5c+8.9d=$	$-.033$	$-.016$	40
	$\geqq 0.070$	.110	$a+5.00b+1.7c+4.8d=$	$-.036$	$-.110$	24
A *g*	$\leqq 0.025$	.017	$a+5.00b-1.1c+17.9d=$	$+.061$	$+.105$	14

TABLE III—*Continued*

Spectrum and Position	Limits of P. M.	Average P. M.	Equations of Condition	No. of Stars
A *g*	.026 to 0.054	0.039	$a+5.00b+0.3c+9.9d=-.020$ resp. -0.003	28
	≧ 0.055	.085	$a+5.00b+1.4c+5.6d=+.057$ $-.012$	16
A2 *e*	≦ 0.025	.018	$a+5.00b-1.0c+17.2d=+.038$ $+.101$	12
	.026 to 0.049	.035	$a+5.00b+0.0c+10.5d=+.103$ $+.142$	19
	.050 to 0.089	.066	$a+5.00b+1.0c+6.8d=+.059$ $+.038$	22
	≧ 0.090	.133	$a+5.00b+2.2c+4.3d=+.094$ $+.014$	13
A2 *g*	≦ 0.025	.0105	$a+5.00b-1.8c+25.6d=+.103$ $+.156$	12
	.026 to 0.059	.036	$a+5.00b+0.4c+10.6d=+.127$ $+.147$	13
	≧ 0.060	.087	$a+5.00b+1.5c+5.6d=+.057$ $-.013$	13
A3 *e*	≦ 0.030	.017	$a+5.00b-0.8c+18.9d=+.104$ $+.161$	11
	.031 to 0.079	.058	$a+5.00b+0.8c+7.4d=+.154$ $+.157$	12
	≧ 0.080	.143	$a+5.00b+2.3c+4.0d=+.125$ $+.050$	9
A3 *g*	≦ 0.035	.027	$a+5.00b-0.6c+12.4d=+.259$ $+.323$	4
	.036 to 0.069	.053	$a+5.00b+0.9c+8.1d=+.147$ $+.153$	7
	≧ 0.070	.095	$a+5.00b+1.1c+4.9d=-.028$ $-.089$	5
A5 *e*	≦ 0.040	.0225	$a+5.00b-0.9c+14.5d=+.169$ $+.246$	6
	.041 to 0.109	.084	$a+5.00b+1.4c+5.7d=+.086$ $+.091$	19
	≧ 0.110	.153	$a+5.00b+2.2c+3.7d=+.332$ $+.289$	13
A5 *g*	≦ 0.035	.021	$a+5.00b-1.0c+15.2d=+.132$ $+.187$	9
	.036 to 0.079	.058	$a+5.00b+0.7c+7.2d=+.236$ $+.264$	9
	≧ 0.080	.120	$a+5.00b+2.0c+4.5d=+.242$ $+.185$	9
F *e*	≦ 0.051	.034	$a+5.00b+0.5c+11.5d=+.324$ $+.356$	4
	.052 to 0.149	.096	$a+5.00b+1.6c+5.2d=+.280$ $+.293$	19
	≧ 0.150	.231	$a+5.00b+2.9c+2.8d=+0.203$ $+.179$	15
F *g*	≦ 0.040	.021	$a+5.00b-0.9c+15.2d=+.247$ $+.267$	6
	.041 to 0.099	.072	$a+5.00b+1.3c+6.5d=+.294$ $+.302$	5
	≧ 0.100	.147	$a+5.00b+2.3c+4.0d=+.307$ $+.373$	7
F5	≦ 0.050	.028	$a+5.00b-0.1c+12.7d=+.320$ $+.300$	5
all	.051 to 0.199	.120	$a+5.00b+1.9c+4.3d=+.345$ $+.363$	13
	≧ 0.200	.315	$a+5.00b+3.2c+2.2d=+.404$ $+.396$	17
G	≦ 0.050	.027	$a+5.00b-0.8c+12.0d=+.644$ $+.606$	7
all	.051 to 0.199	.139	$a+5.00b+2.1c+4.0d=+.536$ $+.544$	6
	≧ 0.200	.501	$a+5.00b+4.0c+1.6d=+.426$ $+.438$	17
G5 *e*	≦ 0.050	.036	$a+5.00b+0.0c+10.5d=+.620$ $+.603$	11
	.051 to 0.129	.092	$a+5.00b+1.7c+5.5d=+.626$ $+.607$	4
	≧ 0.130	.792	$a+5.00b+4.7c+1.1d=+.625$ $+.651$	10
G5 *g*	≦ ≦ 0.050	.0215	$a+5.00b-1.0c+14.7d=+.712$ $+.701$	8
	≧ ≧ 0.051	.628	$a+5.00b+4.5c+1.4d=+.615$ $+.627$	7
K *e*	≦ 0.035	.022	$a+5.00b-0.7c+14.9d=+.783$ $+.766$	34
	.036 to 0.069	.048	$a+5.00b+0.7c+8.5d=+.854$ $+.842$	25
	.070 to 0.139	.101	$a+5.00b+1.4c+4.9d=+.834$ $+.821$	38
	≧ 0.140	.377	$a+5.00b+3.4c+1.9d=+.785$ $+.833$	29
K *g*	≦ 0.035	.018	$a+5.00b-1.2c+16.9d=+.908$ $+.895$	23
	.036 to 0.099	.062	$a+5.00b+0.8c+6.9d=+.800$ $+.791$	26
	≧ 0.100	.291	$a+5.00b+3.1c+2.3d=+.749$ $+.772$	22
K5	≦ 0.035	.027	$a+5.00b-0.5c+11.5d=+1.267$ $+1.262$	5
all	.036 to 0.099	.056	$a+5.00b+0.5c+7.4d=+1.067$ $+1.047$	8
	≧ 0.100	.136	$a+5.00b+2.0c+4.0d=+0.954$ $+0.991$	5
Ma	≦ 0.035	.023	$a+5.00b-0.7c+14.3d=+1.163$ $+1.154$	7
all	.036 to 0.069	.043	$a+5.00b+0.1c+8.85d=+1.236$ $+1.229$	1
	≧ 0.070	.139	$a+5.00b+1.9c+4.0d=+1.010$ $+1.025$	5

TABLE IV

MAURY STARS*

Spectrum and Position	Limits of P. M.	Average P. M.	Equations of Condition	No. of Stars
IV = B3 all	VII ≦ 0".029	0".015	$a+4.50b-1.4c+19.2d=-0.007$ resp. -0.004	15
	IV V VI ≦ 0.030	.043	$a+4.50b-0.5c+8.2d=+.081$ $+.075$	8
V = B5 all	V VI ≦ 0.033	.022	$a+4.50b-1.3c+13.7d=+.082$ $+.094$	9
	IV V VI ≦ 0.034	.049	$a+4.50b-0.1c+7.8d=+.128$ $+.116$	9
VII = A *e*	VI VII ≦ 0.040	.028	$a+4.50b-0.7c+11.9d=+.065$ $+.113$	9
	IV V VI ≦ 0.041	.120	$a+4.50b+1.0c+4.0d=+.084$ $+.041$	10
VII = A *g*	VI VII ≦ 0.035	.018	$a+4.50b-1.6c+15.9d=+.081$ $+.121$	6
	IV V VI ≦ 0.036	.071	$a+4.50b+0.4c+5.8d=+.101$ $+.061$	6
VIII = A *e*	VI VII ≦ 0.040	.028	$a+4.50b-0.6c+11.9d=+.118$ $+.196$	7
	0".041 to 0.089	.070	$a+4.50b+0.2c+5.8d=+.046$ $+.055$	7
	IV ≦ 0.090	.154	$a+4.50b+1.2c+3.3d=+.168$ $+.106$	10
VIII = A *g*	VI VII ≦ 0.043	.024	$a+4.50b-0.6c+13.9d=+.101$ $+.146$	5
	IV V VI ≦ 0.044	.094	$a+4.50b+1.2c+5.0d=+.216$ $+.171$	5
IX = A2 *e*	VI VII ≦ 0.045	.031	$a+4.50b-0.7c+10.8d=+.313$ $+.410$	3
	0.046 to 0.084	.062	$a+4.50b+0.2c+6.4d=+.112$ $+.140$	2
	IV ≦ 0.085	.133	$a+4.50b+1.3c+3.8d=+.261$ $+.226$	10
IX = A2 *g*	VI VII ≦ 0.040	.026	$a+4.50b+0.0c+12.2d=+.254$ $+.308$	5
	IV V VI ≦ 0.041	.078	$a+4.50b+0.4c+5.4d=+.213$ $+.168$	6
XIV = G5 all	VI VII ≦ 0.030	.018	$a+4.50b-1.5c+16.1d=+.937$ $+.927$	10
	IV V VI ≦ 0.040	.172	$a+4.50b+1.7c+3.2d=+.651$ $+.661$	10
XV = K *e*	VI VII ≦ 0.055	.032	$a+4.50b-0.5c+10.9d=+1.047$ $+1.024$	9
	0.056 to 0.149	.100	$a+4.50b+0.9c+4.5d=+0.944$ $+0.920$	27
	IV ≦ 0.150	.316	$a+4.50b+2.7c+2.0d=+0.896$ $+0.930$	25
XV = K *g*	VI ≦ 0.050	.023	$a+4.50b-0.8c+13.9d=+1.076$ $+1.057$	8
	0.051 to 0.119	.090	$a+4.50b+0.5c+4.8d=+0.711$ $+0.689$	6
	IV ≦ 0.120	.394	$a+4.50b+2.5c+1.9d=+0.883$ $+0.904$	14
XVI = K5 all	VI ≦ 0.040	.026	$a+4.50b-0.8c+12.5d=+1.116$ $+1.098$	4
	0.041 to 0.069	.062	$a+4.50b+0.2c+6.5d=+1.149$ $+1.133$	4
	IV ≦ 0.070	.119	$a+4.50b+0.9c+4.0d=+1.213$ $+1.227$	10
XVII = Ma all	V VI ≦ 0.045	.036	$a+4.50b-0.2c+10.1d=+1.333$ $+1.327$	7
	IV V VI ≦ 0.046	.124	$a+4.50b+1.4c+4.1d=+1.307$ $+1.315$	6
XVIII = Ma all	VI ≦ 0.044	.026	$a+4.50b-0.7c+12.8d=+1.261$ $+1.252$	6
	IV ≦ 0.045	.115	$a+4.50b+1.3c+4.3d=+1.382$ $+1.387$	10

* The identification of the spectral classes of Miss Maury with those now generally adopted at Harvard has been taken from *Harvard Annals*, **28**, 145

5. *Corrections to second members of equations of condition.*— For the second members of the above equations two different numbers have been given. The first is the mean of the values of *Phot.* − *Vis.* just as they were taken from *Harvard Annals*, **50.** The second is the same quantity corrected for a somewhat curious source of systematic error, due to the accidental uncertainties in the determination of the class of spectrum. Happily the influence of this error is considerably lessened by the fact that it affects the results for the stars of the spectral classes B to F in one direction; those of the other classes, in the opposite direction. The error alluded to depends on the following facts:

a) The degree of redness regularly increases for stars arranged in the order B, A, F, G, K, M.

b) The average proper motion of the stars thus arranged begins by increasing, passes through a maximum for the stars of the spectrum G, and then diminishes.

The first of these facts is clearly shown by Table XII given farther on. The second is exhibited by Table VII which is derived from Tables V and VI.

TABLE V

AVERAGE PROPER MOTIONS—CANNON STARS

Spectrum	Extra-galactic			Galactic		
	Mean Mag.	Mean P. M.	No.	Mean Mag.	Mean P. M.	No.
Oe5	....	.....	...	5.49	0".034	4
B	....	.....	...	5.32	.015	5
B1	4.29	0".011	1	5.42	.080	1
B2	....	.....	...	4.84	.018	8
B3	5.07	.022	21	5.07	.020	64
B5	4.95	.025	17	5.04	.023	23
B8	5.21	.031	34	5.22	.026	26
B9	5.22	.036	15	5.28	.024	14
A	5.24	.057	84	5.19	.046	58
A2	5.15	.062	66	5.33[5]	.046	38
A3	5.34	.068	32	4.96	.060	16
A5	5.09	.098	38	5.03	.066	27
F	5.21	.143	38	5.24	.084	18
F5	5.21	.221	25	4.60	.153	10
G	4.89	.318	19	4.85	.320	11
G5	5.10	.347	25	4.97	.305	15
K	5.03	.133	126	4.97	.119	71
K2	5.50	.084	2	4.44	.098	2
K5	4.87	.084	12	4.91	.043	6
Ma	4.92	.083	10	5.23	.023	3
Mb	5.13	.049	5	5.38	.032	2

TABLE VI

AVERAGE PROPER MOTIONS—MAURY STARS

Spectrum	Extra-galactic			Galactic		
	Mean Mag.	Mean P. M.	No.	Mean Mag.	Mean P. M.	No.
I = Oe5...	....	.....	...	4.60	0".025	5
II = B.....	....	.....	...	4.66	.010	5
III = B2....	3.25	0".028	4	4.15	.020	8
IV = B3....	4.03	.009	3	4.76	.027	20
IV = B3....	4.35	.018	4	4.44	.031	10
V = B5....	4.16	.037	11	4.70	.033	7
VI = B8....	4.84	.072	10	4.83	.047	6
VII = A.....	4.36	.076	19	4.28	.045	12
VIII = A.....	4.14	.092	24	4.88	.059	10
IX = A2....	4.28	.103	15	4.71	.054	11
X = A5....	4.28	.117	6	4.15	.172	4
XI = F.....	4.21	.168	7	4.48	.089	5
XI = F.....	4.87	.235	3	4.25	.218	4
XII = F5....	4.45	.251	15	3.90	.266	9
XIII = F8....	4.48	.436	16	4.38	.478	5
XIV = G.....	4.00	.398	11	4.27	.089	3
XIV = G5....	4.41	.107	15	4.32	.060	5
XV = K.....	4.24	.178	61	4.16	.223	28
XV = K2....	3.26	.034	5	....	....	...
XVI = K5....	4.21	.092	16	3.65	.038	2
XVII = Ma....	4.58	.056	10	5.00	.078	3
XVIII = Ma....	4.67	.094	11	4.52	.057	5
XIX = Mb....	4.25	.046	4	4.45	.082	4

Summarizing these results and reducing at the same time the proper motions to what they would have been had the average magnitudes been 4.8 throughout, I find the quantities given in Table VII. *Astr. Nachr.*, No. 3487, contains sufficient data for the small reduction involved. No smoothing was applied.

TABLE VII

AVERAGE PROPER MOTIONS FOR STARS OF MAGNITUDE 4.8*

Spectrum	Extra-galactic	Galactic
B.....	...	0".023
B5.....	0".032	.028
A.....	.066	.050
A5.....	.109	.084
F.....	.167	.111
F5.....	.244	.167
G.....	.315	.262
G5.....	.261	.245
K.....	.146	.144
K5.....	.079	.039
M.....	.069	.058

* It has been maintained that the phenomenon of the Milky Way does not show in the stars of the Second Type. The present table indicates clearly that this view is incorrect. For the Second Type, as well as for the First Type, the average proper motion in the Milky Way is smaller than elsewhere. A still stronger proof has already been given. Cf. *Verslagen Kon. Ak. Amsterdam*, 1893.

From the data in Tables VII and XII the necessity of an effect of the kind alluded to above is readily seen. Take, for instance, the stars given as belonging to the spectral class A. Owing to the uncertainties in the classification, some will be included which in reality belong to the classes B9, B8, etc. These classes are bluer (Table XII) and at the same time have generally smaller proper motions (Table VII) than the A stars. The consequence will be that, owing to the admixture of B9, B8, etc., stars, those given as A, and *particularly those having the smaller proper motions*, will on the average be found *too blue*. On the other hand, owing to the same uncertainty in the determination of the spectrum, there will be an admixture of the redder classes of stars A2, A3. . . . . But these, having generally larger proper motions, *will be found preponderatingly among the A stars of greater proper motions*. Therefore, of the stars given as belonging to Class A, those having the larger proper motions will on the average be found *too red*.

We must thus find the increase in redness with diminishing proper motion, that is, with increasing distance, *too small* for the stars of Class A. In other words, we will find too small a value of d. For the stars K, M, for which the proper motion *diminishes* with increasing redness, the reverse will hold.

6. *On the probable error of the classification of spectrum.*—If we knew the probable error r in a determination of the spectral class, the correction necessary to allow for the error in question could readily be computed. But a direct estimate of the real value of the probable error seems hardly possible. I tried to obtain it in two or three different ways, but found the following, utterly discordant results:

$$\left.\begin{array}{l} \text{From 63 stars common to Miss Maury and Miss Cannon } r=\pm 0.07 \\ \text{From 209 stars carefully observed with 8-inch instruments and compared with the observations of Misses Maury and Cannon (with 11-inch and 13-inch instruments)}^1 \quad r=\pm 0.24 \end{array}\right\} \quad (10)$$

The unit in which r is expressed is the interval between Classes B and A, A and F, G and K, K and M.

[1] Taken from *Harvard Annals*, **50**, 234–237. It may be well to quote the following words of Professor Pickering (*ibid.*, **50**, 15): "For the bright stars the larger instruments, and greater dispersion undoubtedly are preferable. For the faint stars the spectra are better marked on photographs taken with the smaller instruments and the presence of many adjacent spectra diminishes the danger of error in identification."

A comparison of the results of Misses Cannon and Maury with the Potsdam visual catalogue was not carried through rigorously, but must yield a probable error *very considerably* greater still. Divergences of 1.5 and even 2.0 are not rare.[1] The first value of r must be too small, not only because it depends for the greater part on stars which are both bright and shown on a large number of plates, but mainly because stars for which the photographic spectra are really practically identical may still differ somewhat in the visual regions which are not included in the Harvard observations. This consideration holds also for the second value, but on the other hand this is probably too great on account of the smaller aperture of the instruments involved. (Compare footnote, however.)

In the absence of an entirely reliable direct determination of r, I have thought that the best method would be to include the probable error among the unknowns of the problem. It may be determined from the fact that the stars of spectra B to F and those of spectra K to M must give the same value for d. The easiest way to accomplish this seems to be to make the computations on two different hypotheses.

a) Supposing the probable error in question to be zero, that is, supposing that the classification of the spectra is absolutely perfect.

b) Assuming for the probable error a certain, arbitrarily chosen value. For this purpose I chose the second of the above values, viz:

$$r = \pm 0.24 \qquad (11)$$

Comparing the resulting values for the constant d furnished by the stars of spectra B to F and by those of spectra K to F we can easily find, by interpolation or extrapolation, the value of r which will make the difference disappear.

7. *Computation of the corrections required by the quantities Phot.—Vis. on account of the accidental errors in the classification of spectra, assuming* $r = \pm 0.24$.—The following two tables furnish the means for this computation.

Table VIII has been derived from the stars of known motion contained in Miss Cannon's catalogue. It must be considered valid therefore for objects between magnitudes 4.5 and 6.0, that is, for stars of average magnitude 5.1 or 5.2. It has also been used for the

[1] *Ibid.*

TABLE VIII

NUMBER OF PROPER MOTIONS BETWEEN DETERMINED LIMITS (IN FRACTIONS OF WHOLE)

Limits of μ	B_3	B_5	B8	B_9	A	A_2	A_3	A_5	F	F_5	G	G_5	K	Mab
0".000 to 0".020	0.53^5	0.49	0.39^5	0.40	0.19	0.18	0.14	0.07	0.03	0.03	0.07^5	0.12	0.14	0.20
.021 to .040	.36	.38^5	.39	.41	.30	.24	.19	.10	.07	.05	.11^5	.23	.16	.25
.041 to .060	.09	.10^5	.14	.13	.22^5	.21	.21	.15	.10	.05	.09^5	.16	.14^5	.20
.061 to .080	.01	.02	.04^5	.05	.13	.16	.18	.18	.14	.06	.07^5	.09	.12	.13
.081 to .100	....	....	.02^5	.01	.07	.09	.12	.16	.14	.08	.06	.06	.10	.08
.101 to .120	....	....	.00^5	...	.03^5	.05	.07	.12	.11	.09	.05	.04	.07^5	05
.121 to .140	....	....	....	....	.02	.02^5	.04^5	.08	.07	.06^5	.05	.03	.06^5	.03
.141 to .160	....	....	....	....	.01	.02	.02^5	.05	.06	.05^5	.04	.03	.04	.02
.161 to .180	....	....	....	....	.01	.01^5	.01^5	.03	.06	.05	.03^5	.02^5	.03	.01^5
.181 to .200	....	....	....	....	.00^5	.01	.0^5	.02	.05	.05	.03	.02	.02	.01
.201 to .220	....	....	....	....	.00^5	....	....	.02	.04	.04	.03	.02	.01^5	.01
.221 to .240	....	....	....	....	....	....	....	.01	.03	.04	.02^5	.02	.01	.00^5
.241 to .260	....	....	....	....	....	....	....	.01	.03	.04	.02^5	.01^5	.01	....
.261 to .280	....	....	....	....	....	....	....	....	.03	.04	.02	.01^5	.00^5	....
.281 to .300	....	....	....	....	....	....	....	....	.02	.04	.02	.01	.00^5	....
.301 to .320	....	....	....	....	....	....	....	....	.01	.03	.02	.01	.00^5	....
.321 to .340	....	....	....	....	....	....	....	....	.01	.03	.02	.00^5	.00^5	....
.341 to .360	....	....	....	....	....	....	....	....	....	.03	.01^5	.00	.00^5	....
.361 to .380	....	....	....	....	....	....	....	....	....	.03	.01^5	.00	.00	....
.381 to .400	....	....	....	....	....	....	....	....	....	.03	.01^5	.00	.00	....
.401 to .420	....	....	....	....	....	....	....	....	....	.02	.01	.00	.00	....
.421 to .440	....	....	....	....	....	....	....	....	....	.02	.01	.00	.00	....
≧0.441	....	....	....	....	....	....	....	....	....	.03	.15	.10	.04^5	....
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

stars of Miss Maury, which are generally somewhat brighter; but an appreciable error cannot have been introduced in this way. The values of the table have been slightly smoothed by empirical curves. Before using them, the table was extended by interpolation to the intermediate classes of spectra. The sudden change between the values for B8 and B9, and A is rather remarkable.

The meaning of Table IX is that of the stars observed as belonging, say, to Class A, the fraction 0.113 belongs really to this class; the fraction 0.106 belongs to B9 and another equal fraction to A1. Similarly the fraction 0.097 belongs to B8 and another equal fraction

TABLE IX

MIXTURE OF SPECTRAL CLASSES IN WHAT HAS BEEN OBSERVED AS SPECTRUM X (ASSUMING $r = \pm 0.24$)

Spectrum	Fraction	Spectrum	Fraction
X±1.1	0.0015	X±0.5	0.042
X±1.0	.002	X±0.4	.059
X±0.9	.005	X±0.3	.079
X±0.8	.009	X±0.2	.097
X±0.7	.016	X±0.1	.106
X±0.6	.027	X	.113

to A2, and so on. The table has been computed assuming the ordinary law of error and the value (11) for r.

An example will serve to illustrate the use of these tables. Take the stars observed by Miss Cannon as belonging to Class F, extra-galactic. In Table III they have been divided into three groups, having their proper motions included between the limits:

$$\leqq 0''.051; \quad 0''.052 \text{ to } 0''.149; \quad \geqq 0''.150 \qquad \text{(A)}$$

According to Table IX

the fraction 0.0015 will really belong to Class B9
" " 0.002 " " " " " A

etc.

According to Table VIII

of the stars B9 the fractions		0.87	0.13	0.00
" " " A " "		0.60	0.37	0.03

etc.

will have their proper motion between the limits (A). Therefore,

of the stars observed as belonging to spectrum F, the following fractions will really belong to the Classes B9, A. . . . .

	P. M. $\leqq$ 0".051	0".052 to 0".149	$\geqq$ 0".150	Table XII Average Value of *Phot.* − *Vis.*
To B9	$0.001^5 \times 0.87$	$0.001^5 \times 0.13$	$0.001^5 \times 0.00$	−0.04
" A	0.002×0.60	0.002×0.37	0.002×0.03	−0.03
	etc.	etc.	etc.	etc.

From this table, completed, we find that, of the stars observed as F

those having P. M.	will have average value of *Phot.* − *Vis.*
$\leqq$ 0".051	+0.211 Mag.
0".052 to 0.149	+0.230
$\geqq$ 0.150	+0.267

In order to reduce the observed values of *Phot.* − *Vis.* to the case of perfect freedom of error in classification, we may therefore apply corrections such as

P. M.	Corrections to *Phot.* − *Vis.*	No.
$\leqq$ 0".051	+0.032 Mag.	4
0".052 to 0.149	+0.013	19
$\geqq$ 0.150	−0.024	15
Mean	0.000	

Computing the corrections for all the different classes of spectra in the same way and applying them to the observed values of *Phot.* − *Vis.* we obtain the corrected values of the second members in the equations of Table III and IV.

8. *First solution of the equations of condition.*—Turning now to the solution of the equations of condition, it is easily seen that the two quantities c and d cannot both be determined with any degree of accuracy owing to the fact that their coefficients vary nearly proportionally. For this reason I have derived the value of d as a function of c. In this derivation I have used for each class of spectrum only two of the equations given in Tables III and IV. In cases where the number is three or four, either the extreme ones alone were used, or else the last two were first combined in accordance with their weight. For the stars of Miss Maury the latter was done in every case; for those of Miss Cannon only in the case of A2 extra-galactic, A5 extra-galactic, and F galactic.

The results are as follows:

	Phot.−*Vis.* uncorrected	*Phot.*−*Vis.* corrected ($r=\pm 0.24$)
Miss Cannon	$d=+0.0025+0.246c\pm 0.0010$	$d=+0.0055+0.246c\pm 0.0010$
Miss Maury	$d=+0.0079+0.223c\pm 0.0027$	$d=+0.0104+0.223c\pm 0.0025$
Mean	$d=+0.00315+0.243c\pm 0.0009$	$d=+0.0062+0.243c\pm 0.0009$ (12)

9. *Value of c.*—From this result a fairly trustworthy value for d may be derived, if it proves feasible to obtain the value of c independently, or approximately so, from the other unknowns of the problem. The method which promises some approach to such an independent determination was indicated in my first paper.[1] We have simply to compare stars of the same spectrum, the same apparent magnitude, but with different, though in all cases considerable, proper motions or parallaxes. Such suitable material of this kind as I have been able to collect is given in what follows:

a. Spectrum F5

Within the limits of proper motion $\mu > 0''.400$ and $0''.068 < \mu < 0''.150$ I find the following average values:

Mean Mag.	P. M.	*Phot.*−*Vis.*	Corrected	No. of Stars
4.73	0".467	+0.607	+0.556	7
4.80	0.115	+0.378*	+0.378	16

* One star giving an exceptionally discordant value of *Phot.*−*Vis.* was rejected.

The corrections to *Phot.*−*Vis.* were computed exactly as in Section 5, only for r was taken the value +0.16 in accordance with what is found farther on. As it is only the difference of the corrections which is of importance, I have subtracted a number such that the correction for the stars of smaller proper motion become zero.

Computing the corresponding values of π, M, Δ, we find from them the equations of condition:

$$a+4.73b+3.69c+1.61d=+0.556$$
$$a+4.80b+1.56c+4.44d=+0.378$$

whence, if, in accordance with (9), we take $b=-0.31$,

$$c=+0.073+1.33d\pm 0.031 \text{ (P. E.)}^2 \qquad (13)$$

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 31.

[2] The probable error has been computed by means of the probable error ± 0.148 found farther on for one observation of *Phot.*–*Vis.*

b. *Spectrum G*

The following stars, all extra-galactic, have fairly large, directly determined parallaxes:

Star	Mag. Harv.	π	Authority	M	Δ	*Phot.−Vis*
ξ *Ursae Majoris*	3.86	0".178	Elkin, Chase	5.11	0.56	+0.81
1830 Groombridge	6.46	0.148	Various	7.31	0.68	+0.17
β *Comae*	4.32	0.118	Peter, Chase	4.68	0.85	+0.84
5 *Serpentis*	5.18	0.208	Elkin, Smith	6.77	0.48	+0.16
ζ *Herculis*	3.00	0.178	Smith	4.25	0.56	+0.93
Mean	4.56			5.62	0.63	+0.582

With these I compare the results for the extra-galactic Cannon and Maury stars having proper motions between 0".090 and 0".230, I find eight such stars in my collection, giving the average values:

Mag.=4.79 P. M.=0".160 *Phot.−Vis.* =+0.625

For these values we find by *Groningen Publications*, No. 8:

π=0.0275, consequently M=1.99, Δ=3.6

The equations of condition for the two groups therefore become:

$$a+4.56b+5.62c+0.63d=+0.582$$
$$a+4.79b+1.99c+3.6\ d=+0.544$$

The corrections to *Phot.−Vis.* are nearly insensible. Therefore, introducing the value $b=-0.296$ from (9), we get:

$$c=-0.029+0.82d\pm0.024 \text{ (P. E.)} \qquad (14)$$

c. *Spectrum G5*

The following stars have relatively large, directly determined parallaxes:

Star	Mag.	π	Authority	M	Δ	*Phot.−Vis.*	Position
μ *Cassiopeiae*	5.26	0".108	Peter, Schlesinger	5.42	0.93	+0.38	*g*
Piazzi 2^h, 123	5.92	0.148	Elkin, Smith	6.77	0.68	+0.02	*e*
o_2 *Eridani*	4.48	0.178	Gill	5.73	0.56	+0.71	*e*
61 *Virginis*	4.80	0.157	Flint	5.78	0.64	+0.80	*e*
μ *Herculis*	3.50	0.106	Chase, Carn.-Camb.*	3.63	0.94	+0.91	*e*
Mean	4.79			5.47	0.75	+0.564	
					corrected	+0.594	

* By Carn.-Camb. (Carnegie-Cambridge) are designated the parallaxes obtained by Russell, who for the work obtained a grant from the Carnegie Institution, and Hinks, Cambridge. I take this occasion to thank these gentlemen, and also Dr. Schlesinger, for the very kind way in which they placed their unpublished observations at my disposal.

With these I compare the stars of Miss Cannon and Miss Maury having proper motions between 0".090 and 0".150. They are distributed over the galactic and extra-galactic regions about in the same proportion as the above stars. The averages are:

Mag.=4.99 P. M.=0".114 *Phot.*−*Vis.*=+0.646

with which correspond

π=0.0220 M=1.70 Δ=4.5

The equations of condition for the two groups, then, are:

$a+4.79b+5.47c+0.75d=+0.594$ (5 stars)
$a+4.99b+1.70c+4.50d=+0.646$ (10 stars)

Therefore with the value −0.296 of b,

$$c=-0.029+0.99d\pm0.022 \text{ (P. E.)} \qquad (15)$$

d. Spectrum K

The following stars have relatively large, well-determined parallaxes:

Star	Mag.	π	Authority	M	Δ	*Phot.*−*Vis.*	Position
54 *Piscium*	6.08	0".148	Chase	6.93	0.68	+0.44	*e*
Mayer 20	5.82	0.168	Chase	6.95	0.60	+0.32	*e*
τ Ceti	3.65	0.318	De Sitter	6.16	0.31	+0.98	*e*
p Ophiuchi	4.07	0.158	Krüger	5.07	0.63	+1.00	*g*
Bradley 3077	5.65	0.138	Peter	6.35	0.72	+0.44	*g*
Mean	5.05			6.29	0.59	+0.636	

To these may be added the stars, not contained in the preceding list, having proper motions exceeding 0".50. I find:

	Mag.	μ	*Phot.*−*Vis.*	No. Stars
Extra-galactic	4.73	0".749	+0.77	9
Galactic	4.07	1.190	+0.99	5
Mean	4.49	0.906	+0.851	14

to which correspond:

π=0".101 M=4.51 Δ=1.00

In order not to diminish the coefficients of c too much I combine the two results with equal weight and obtain the equation of condition:

$a+4.77b+5.40c+0.80d=+0.744$, or corrected=+0.819 (19 stars)

With these I compared the following stars of Miss Maury and Miss Cannon:

	μ	Mean Mag.	Mean P. M.	*Phot.*−*Vis.*	No. Stars
Cannon extra-gal	0″.070 to 0″.139	4.94	0″.101	+0.854	38
Cannon gal.....	.090 to .150	5.04	.112	+0.809	15
Maury extra-gal.....	.090 to .150	4.31	.121	+1.023	15
Maury gal.....	.090 to .150	3.89	.121	+1.044	7

Combining the values for the two observers and then giving the weights 3 and 2, respectively, to the extra-galactic and galactic stars, I find:

Mean Mag. 4.73 P. M.=0″.110 *Phot.*−*Vis.*=+0.893 (75 stars)

with which correspond

$$\pi=0''.0221 \qquad M=1.45 \qquad \Delta=4.52$$

and the equation of condition:

$$a+4.73b+1.45c+4.52d=+0.893 \text{ (75 stars)}$$

Subtracting the two equations and introducing the value of b we get:

$$c=-0.015+0.94d\pm0.012 \text{ (P. E.)} \qquad (16)$$

e. Spectrum K5

The only star of considerable parallax that has been measured is 61 *Cygni* for which

Mag.=5.9 $\pi=0''.308$ $M=8.34$ $\Delta=0.32$
Phot.−*Vis.*=+1.11, or corrected +1.176

Therefore

$$a+5.9b+8.34c+0.32d=+1.176 \text{ (1 star)}$$

In order to match the apparent magnitude as nearly as possible (so that the value of b may not have an undue influence) I take for comparison only the stars of Miss Cannon, and of these the stars with proper motion exceeding 0″.100, which give:

$$a+5.03b+2.0c+4.0d=+0.944 \text{ (5 Stars)}$$

Subtracting the two equations and taking $b=-0.336$ we get:

$$c=+0.083+0.58d\pm0.25 \text{ (P. E.)} \qquad (17)$$

Collecting the values of c:

F5.....$c=+0.073 \quad +1.33d \pm 0.031$
G.....$c=- \ .029 \quad +0.82d \pm 0.024$
G5.....$c=- \ .029 \quad +0.99d \pm 0.022$
K.....$c=- \ .015 \quad +0.94d \pm 0.012$
K5.....$c=+ \ .083 \quad +0.58d \pm 0.025$

Mean.....$c=-0.0004+0.92d \pm 0.0086$ (P. E.) (18)

The agreement of the separate values is bad and the probable error should perhaps be increased somewhat in order to represent the real accuracy of the result.

If we combine the value (18) with what we find from (12) by interpolation for $r=\pm 0.16$ (which we shall presently find for this quantity) we get:

$$c=+0.0057 \pm 0.0111$$

TABLE X

VALUES OF d FOR THE CANNON STARS

Spectrum and Position	$r=0.00$	$r=\pm 0.24$	No. of Stars	Weight	$r=\pm 0.16$
B3 *e*.....	+0.0038	+0.0037	21	520	+0.0037
B3 *g*.....	− .0019	− .0013	64	830	− .0015
B5 *e*.....	+ .0019	+ .0036	17	160	+ .0030
B5 *g*.....	− .0010	+ .0006	23	210	+ .0001
B8 *e*.....	+ .0021	+ .0062	34	530	+ .0048
B8 *g*.....	− .0106	− .0071	26	565	− .0083
B9 *e*.....	− .0017	+ .0059	15	145	+ .0034
B9 *g*.....	− .0007	+ .0040	14	290	+ .0024
A *e*.....	+ .0054[5]	+ .0136	84	2130	+ .0109
A *g*.....	+ .0017	+ .0103	58	1150	+ .0074
A2 *e*.....	− .0027	+ .0081	66	1230	+ .0045
A2 *g*.....	+ .0015	+ .0067[5]	38	2660	+ .0050
A3 *e*.....	− .0023	+ .0052	32	1270	+ .0027
A3 *g*.....	+ .0367	+ .0528	16	130	+ .0474
A5 *e*.....	− .0094	+ .0012	38	490	− .0023
A5 *g*.....	− .0117	− .0021	27	560	− .0053
F *e*.....	+ .0153	− .0217	38	240	+ .0196
F *g*.....	− .0119	− .0085	18	430	− .0096
F5 all.....	− .0080	− .0093	35	420	− .0089
G all.....	+ .0203	+ .0156	30	550	+ .0172
G5 *e*.....	− .0005	− .0052	25	460	− .0036
G5 *g*.....	+ .0073[5]	+ .0056	15	660	+ .0062
K *e*.....	− .0013	− .0050	126	3060	− .0038
K *g*.....	+ .0108	+ .0087	71	2550	+ .0094
K5 all.....	+ .0417	+ .0367	18	140	+ .0384
Ma all.....	+ .0133	+ .0116	13	310	+ .0122
Means					
B3 to F.....	− .0001	+ .0063	629	13540	+ .0042
F5, G, G5.....	+ .0059	+ .0029	90	2090	+ .0039
K, K5, Ma.....	+ .0055	+ .0026	243	6060	+ .0036
Total.....	+ .0020	+ .0050	962	21690	+ .0040

TABLE XI

VALUES OF d FOR THE MAURY STARS

Spectrum and Position	$r=0.00$	$r=\pm0.24$	No. of Stars	Weight	$r=\pm0.16$
IV = B3 all.....	−0.0079	−0.0072	23	210	−0.0074
V = B5 all.....	− .0078	− .0038	18	50	− .0051
VII = A *e*.....	− .0024	+ .0091	19	300	+ .0053
VII = A *g*.....	− .0020	+ .0059	12	310	+ .0037
VIII = A *e*.....	− .0041	+ .0116	24	310	+ .0064
VIII = A *g*.....	− .0129	− .0028	10	200	− .0062
IX = A2 *e*.....	+ .0050	+ .0252	15	110	+ .0185
IX = A2 *g*.....	+ .0060	+ .0206	11	130	+ .0157
XIV = G5 all.....	+ .0222	+ .0205	20	830	+ .0211
XV = K *e*.....	+ .0171	+ .0101	61	520	+ .0124
XV = K *g*.....	+ .0181	+ .0149	28	750	+ .0160
XVI = K5 all.....	− .0116	− .0154	18	200	− .0141
XVII = Ma all.....	+ .0043	+ .0020	13	120	+ .0028
XVIII = Ma all.....	− .0142	− .0159	16	270	− .0153
Means					
B3 to A2.....	− .00365	+ .0070	132	1620	+ .00345
G5.....	+ .0222	+ .0205	20	830	+ .0211
K, K5, Ma.....	+ .00905	+ .0050	136	1860	+ .00635
Total.....	+ .0068	+ .0087	288	4310	+ .0081

It is certainly to be regretted that we cannot derive a value of c having a higher precision. Meanwhile it appears clearly that in the data at present available there is no indication that the value of c is different from *zero* and we will therefore provisionally adopt that value in what follows.

10. *Second solution of the equations of condition.*—With the value $c=0$ I re-solved the equations of condition of Section 4, neglecting none of these equations. Wherever the number of equations exceeded two, the solution was made by least squares, with due regard for the weights. I found the results shown in Tables X and XI.

The unit of weight is in general the weight of one determination of *Phot.*−*Vis.* As, however, it may be doubted whether the parallaxes derived from *Groningen Publications*, No. 8, are quite valid for the B stars, and as, at all events, owing to the smallness of their proper motions, the proportional uncertainty in the amount of this motion, and consequently in the parallaxes, is relatively high, the weight has been divided by *three* for the stars of spectra B3 and B5, and by *two* for the stars of spectra B8 to B9.

According to what has been said in Section 6, we have now to determine the probable error r of one determination of the spectral

class from the condition that the value of d must be the same for stars B to F as for those of spectra K to M. From the means at the bottom of Tables X and XI, we find at once by interpolation that this will be the case

for the Cannon stars when $r=\pm 0.145$
" " Maury " " $r=\pm 0.21$

The relative weights of the two results being about as 5 to 1, we adopt

$$r=\pm 0.16 \qquad (19)$$

with which we get the final values of d inserted in the last column of Tables X and XI.

The comparison of the individual values leads to the probable error of the unit of weight, that is,

Probable error of a single value of Phot.−Vis. $=\pm 0.148$ Mag. (20)

The values obtained from the two tables agree almost perfectly.

The only value of d which, on account of its great weight, seems to call for particular notice is the negative one for the extra-galactic K stars of Miss Cannon. A possible explanation is that just the fact of a somewhat greater faintness in the violet part of the spectrum may have had a tendency to make Miss Cannon place a star in the K5 division. This would at the same time explain the abnormally high value of d for the K5 stars. Taking the divisions K and K5, galactic and extra-galactic, together we get

$$d=+0.0031 \quad \text{weight } 5780$$

which does not differ from the final value more than well may be attributed to accidental errors.

Deriving now the final value of d, in the first instance separately for the galactic and extra-galactic regions, we find:

	Miss Cannon	Miss Maury	Mean
Galactic.....	$d=+0.0044\pm 0.0015$	$d=+0.0101\pm 0.0039$	$d=+0.0051\pm 0.0014$
Extra-galactic..	$d=+0.0027\pm 0.0015$	$d=+0.0097\pm 0.0042$	$d=+0.0035\pm 0.0014$
		Mean	$d=+0.0043\pm 0.0010$

There is thus no reason for the supposition that the selective loss of light is different for galactic and extra-galactic regions. In particular we cannot explain the greater richness in stars of the Milky Way by a smaller space-absorption, for, if anything, this absorption

is greater there than elsewhere. In deriving the values for the separate spectral classes we may thus safely combine the two regions. We get:

Spectrum	Miss Cannon	Miss Maury	Mean
B3 to B9.....	$d=+0.0001\pm0.0026$	$d=-0.0069\pm0.0091$	$d=-0.0004\pm0.0025$
A to A5.....	$d=+\ .0058\pm\ .0015$	$d=+\ .0056\pm\ .0040$	$d=+\ .0058\pm\ .0014$
F to F5.....	$d=-\ .0029\pm\ .0045$		$d=-\ .0029\pm\ .0045$
G to G5.....	$d=+\ .0071\pm\ .0036$	$d=+\ .0211\pm\ .0051$	$d=+\ .0118\pm\ .0029$
K to Ma.....	$d=+\ .0035^5\pm\ .0019$	$d=+\ .0064\pm\ .0034$	$d=+\ .0043\pm\ .0017$
Mean	$d=+0.0040\pm0.0010$	$d=+0.0081\pm0.0023$	$d=+0.0048\pm0.0009$

The agreement of the separate values is scarcely below what we had a right to expect. We may thus adopt provisionally

$$d=+0.0048^1\pm0.0009 \qquad (21)$$

11. *Values of* $a+5.00b$.—We can now derive the values of $a+5.00b$, that is, the average degree of redness, as defined by the quantity *Phot.* $-$ *Vis.* freed from the influence of space-absorption, for stars of the 5th magnitude. These values were necessary for the considerations of Section 5, and have of course an importance apart from the object of the present paper.

With the values $c=0$, $d=+0.0048$, and $r=\pm0.16$, Tables III and IV furnish us at once with the values of $a+5.00b$ for Miss Cannon and $a+4.50b$ for Miss Maury. We reduce the latter value to the former by use of the mean of the values (9) found for the two observers. Having regard to the weight of the different equations we find the results recorded in the table on p. 25.

In parentheses is given the number of stars from which the results are derived. There is no very definite trace of a difference between galactic and extra-galactic regions, and the agreement between the results for the stars of the two observers is satisfactory. We may therefore safely unite the different results. Table XII represents the result of such a combination.

12. *Total loss of light.*—In what precedes we have found the *difference* between the loss for the visual and photographic rays. From this difference we might derive the total loss separately for

[1] As a good partial check on our computations we find, interpolating the value of d for $r=\pm0.16$ in (12) and taking $c=0$, $d=+0.0052\pm0.0009$. The value (21) based on the *whole* of our data is of course to be preferred.

CANNON STARS

SPECTRUM	$a+5.00b$					
	Extra-galactic		Galactic		Mean	
B3	−0.135	(21)	−0.094	(64)	−0.105	(85)
B5	− .129	(17)	− .039	(23)	− .078	(40)
B8	− .114	(34)	− .047	(26)	− .085	(60)
B9	− .093	(15)	− .124	(14)	− .108	(29)
A	− .068	(84)	− .032	(58)	− .053	(142)
A2	+ .030	(66)	+ .029	(38)	+ .029	(104)
A3	+ .077	(32)	+ .080	(16)	+ .078	(48)
A5	+ .152	(38)	+ .169	(27)	+ .159	(65)
F	+ .231	(38)	+ .277	(18)	+ .246	(56)
F5	.....	...	.....	...	+ .348	(35)
G	.....	...	.....	...	+ .476	(30)
G5	+ .594	(25)	+ .625	(15)	+ .606	(40)
K	+ .776	(26)	+ .776	(71)	+ .776	(197)
K5	.....	...	.....	...	+1.054	(18)
Ma	.....	...	.....	...	+1.061	(13)

MAURY STARS

SPECTRUM	$a+5.00b$					
	Extra-galactic		Galactic		Mean	
IV = B3	.....	...	.....	...	−0.137	(23)
V = B5	.....	...	.....	...	− .033	(18)
VII and VIII = A	−0.031	(43)	−0.023	(22)	− .028	(65)
IX = A2	+ .129	(15)	+ .095	(11)	+ .115	(26)
XIV = G5	.....	...	.....	...	+ .593	(20)
XV = K	+ .743	(61)	+ .698	(28)	+ .729	(89)
XVI = K5	.....	...	.....	...	+ .971	(18)
XVII and XVIII = Ma	.....	...	.....	...	+1.181	(29)

the rays of different wave-lengths were it possible to adopt the hypothesis: *that practically all the loss of light is due to scattering.* It is well known that shooting stars must generally be extremely minute bodies. If this is true for the shooting stars visible to the naked eye, it is of course true *a fortiori* for the telescopic ones, the number of which falling daily on the earth has been evaluated by the best authorities at several hundred million. It seems rational to assume that there must be a far greater number of still smaller meteors which escape all observation. As the earth meets these bodies everywhere in its orbit round the sun and in its course through space with the whole of the solar system, there seems reason for supposing that the whole of our stellar system is filled with

meteoric dust. This dust must produce a scattering of light and it seems only natural to attribute the selective loss of light indicated by the present investigation to this cause. For it is well known that the scattering is selective and the stronger, the shorter the wave-length. Meanwhile the larger meteors must intercept the light of different wave-lengths much to the same extent.

In the absence of any reliable data we cannot decide what must be the relative influence of the two causes. If, however, we may assume that they are practically the only causes of loss of light, then our hypothesis must at least furnish us with a *lower limit* for the total

TABLE XII

Phot.—Vis. CORRECTED FOR SPACE-ABSORPTION

Spectrum	$a+5.00b$
	Mag.
B3	−0.11
B5	− .09
A	− .03
A5	+ .16
F	+ .25
F5	+ .35
G	+ .48
G5	+ .60
K	+ .76
K5	+1.01
Ma	+1.10

loss. For, whereas by what precedes we have the *difference* of the loss of light for the visual and photographic rays, the law found by Lord Rayleigh, that the scattering of light is proportional to the fourth power of the wave-length, gives the *ratio* of the two losses.

The photographic magnitudes of the *Draper Catalogue* depend on estimates of brightness at wave-length 0.431 μ. The visual magnitudes depend on the rays of the whole of the visual spectrum. The relative part contributed to the total visual brightness by the light of different wave-lengths has been determined by Professor S. P. Langley in a memoir on "Energy and Vision."[1] Taking the means of the results obtained by the three observers: F. W. V.; B. E. L.; E. M., each including from 2 to 4 days' work, we have

Photometric Values (Normal Spectrum)

Wave-length	0.40μ	0.45	0.50	0.55	0.60	0.65	0.70
Relat. part contributed	0.7	32.1	131.1	111.4	20.8	1.5	0.3

[1] *Memoirs National Academy of Sciences*, **5**.

If we complete these numbers for the intermediate wave-lengths by graphical interpolation and if then we admit that of all these contributions a part is lost which is proportional to the fourth power of the wave-length, we easily find:

$$\frac{\text{Loss of brightness at } \lambda=0.431\mu}{\text{Loss of visual brightness}} = \frac{2.03}{1} \tag{22}$$

On the other hand the loss (21) of 0.0048 magnitudes is equivalent to a loss of the fraction 0.0044 of the total light.

Therefore

$$\left.\begin{array}{l}\text{Loss of brightness at } \lambda=0.431\mu - \text{loss of visual brightness} \\ =0.0044\times\text{total brightness}\end{array}\right\} \tag{23}$$

From (22) and (23) there results at once:

$$\left.\begin{array}{l}\text{Total loss of light over a distance of 32.6 light-years (corresponding to a parallax of 0".1)} \\ \text{for the photographic rays}=0.00867\times\text{total light}=0.00945 \text{ magnitude} \\ \text{“ “ visual “ }=0.00427\times \text{“ “ “ }=0.00465 \text{ “}\end{array}\right\} \tag{24}$$

which values must provisionally be considered as lower limits.

CONCLUDING REMARKS

I. Some objections may be raised against the preceding results:

a) It may be questioned whether for the B stars, having such exceptionally small proper motions (see Table VII), the table for the parallaxes given in *Groningen Publications*, No. 8, is sufficiently accurate. Moreover the observation errors must have a very appreciable influence upon the amount of this proper motion. In the above I have tried to allow for this uncertainty by diminishing the weights. If we reject altogether the values of d furnished by these stars, we find the following averages:

VALUES OF d—MISS CANNON

Spectrum	$r=0.00$	$r=\pm0.24$	Weight	$r=\pm0.14$
A to F.....	+0.0003	+0.0081	10290	+0.00485
F5 to G5.....	+ .0059	+ .0029	2090	+ .0041
K to Ma.....	+ .0059	+ .0026	6060	+ .0038
Total.....	+0.00265	+0.0057	18440	+0.00445

(25)

VALUES OF d—MISS MAURY

Spectrum	$r=0.00$	$r=\pm0.24$	Weight	$r=\pm0.14$	
A to A2.....	−0.0028	+0.0096	1360	+0.0044	
G5.....	+ .0222	+ .0205	830	+ .0212	(26)
K to M.....	+ .00905	+ .0050	1860	+ .0067	
Total.....	+0.0078	+0.0097	4050	+0.0089	

We see that the difference between the first and the third results will disappear, for

$r=\pm0.12$ in the case of Miss Cannon
$r=\pm0.17$ " " " " Miss Maury

so that we shall have to adopt

$$r=\pm0.14. \tag{27}$$

With this we find the results in the last column of (25) and (26). Combining the values for the two observers according to their respective weights we obtain:

$$d=+0.0052^5\pm0.0010 \tag{28}$$

Had we made the solution as a function of c we should have obtained:

$$d=+0.0052^5+0.25c\pm0.0010 \tag{29}$$

Personally I am inclined to think that the rejection of the B stars is justified. But whether this is done or not, we see that the result is hardly changed.

b) The determination of r is indirect. According to what has been said in Section 6, I think that we may safely assume that, at all events, r must lie somewhere between 0.10 and 0.20. For these limits we find from (25) and (26):

	$r=\pm0.10$	Weight	$r=\pm0.20$	Weight
Miss Cannon..	$d=+0.0039$	18440	+0.0052	18440
Miss Maury...	$d=+0.0086$	4050	+0.0094	4050
Mean.....	$d=+0.0047^5$		$+0.0059^5$	

The deviations from the value (28) are well within the limits of probable error.

c) The uncertainties in the distances computed by the table of *Groningen Publications*, No. 8, are relatively great. It is very diffi-

cult to estimate these uncertainties and their influence on the results of the present investigation. As, however, the uncertainties are certainly greater for the stars of very small proper motions, I have made a new solution in which, besides the B stars, all the groups with the smallest proper motion in Tables III and IV were rejected. I thus found:

$$d=+0.0069\pm0.0030 \tag{30}$$

The value of d has changed very little, though of course the weight, now that the stars with the greatest coefficients have been excluded, is enormously diminished.

Further, we must not forget that at least the *reality* of a *positive* value of d (resp. of $d-0.25c$) does not depend on the correctness of the values of *Publications*, No. 8. For what the preceding investigation shows directly and independently of any adopted values of the distance is, that, all other things being equal, the stars are the redder the smaller their proper motion. Therefore, unless we deny that, *ceteris paribus*, the distance must be the greater the smaller the proper motion, d (resp. $d-0.25c$) must be a positive quantity.

d) Uncertainty as to the value of c. This uncertainty undoubtedly is very serious and takes away a considerable part of the value that our determination of d might otherwise have. The values (21), (29), (30) are obtained on the supposition that c must be *zero*. If we may really suppose this, it would seem that not only has the reality of an appreciable selective loss of light in space been demonstrated by what precedes, but that even its amount has been fixed with some approximation. But objection may be taken to such a supposition, and if we do not introduce it, the accuracy is very considerably diminished. For the independent determination of c and d we have in reality, if we reject the B stars, the two equations (29) and (18):

$$\left.\begin{aligned} d&=+0.0052^5+0.25c\ \pm0.0010\\ c&=-0.0004\ +0.92d\ \pm0.0086 \end{aligned}\right\} \tag{31}$$

Their combination leads to:

$$\left.\begin{aligned} c&=+0.0058\pm0.0112\\ d&=+0.0066\pm0.0031 \end{aligned}\right\} \tag{32}$$

The value of d has not been altered very appreciably but its

probable error has been increased *threefold*, that is, it is three times what it would be if we knew that c were necessarily *zero*.

Poor as the determination of d thus becomes, it proves abundantly that such large values as are implied by the results lately brought out by certain investigators must be illusory.

It might be supposed that better results would have been obtained had an equation of condition of the form (4) been written out for each individual star and had the whole of the 1250 equations then been solved by least squares. I do not think so. For *individual* stars the parallax derived from the proper motions and magnitudes must often be very considerably in error. Consequently the coefficients M and Δ of a great many of the equations of condition would have been very greatly in error, and the value to be attached to the results would have been questionable. Moreover, even the *apparent* gain in weight would undoubtedly have been relatively small.

The real cause of the difficulty is in the way in which the *three* unknowns, b, c, d, are bound together. In order to get a good determination of d we must compare stars at widely different distances. If we wish to make the determination independent of c we must take care to compare only stars of about the same average absolute magnitude. But stars of largely different distances will be of equal absolute magnitude only if the apparent magnitudes are very different. Thus the value of b is introduced instead of c.

The difficulty disappears as soon as either b or c is known. It seems not impossible that some method may be devised in which the constant b, or rather the constants b, for we found b different for different classes of spectra, will be zero; that is, a method in which the difference between the measured intensity at two determined points in the spectrum will be independent of the apparent magnitude. Indeed one would have thought a priori such an independence rather probable.

Now that we find, however, that the value of b depends on certain peculiarities in the photographic plates and processes, which may change under changing conditions, it seems safer not to rely on such a vanishing of b, even if we should succeed in obtaining it in certain cases. Nor would it be possible, probably for the same reason, to determine the value or values of b once for all. It seems preferable

to obtain, once for all, the value of c, which is a *cosmical* constant, on the invariability of which we may depend, and the determination of which must be considered as of great importance, even apart from the present problem.

Now such a determination of c independently of the other constants must present no particular difficulty as soon as we take observations expressly for the purpose. For in the comparison of bright and faint stars we can eliminate the constant b altogether by *artificially* diminishing the light of the brighter stars, say by stopping down the aperture.

Take, for instance, a group of stars of the fourth magnitude and a group of stars of the ninth or tenth magnitude, of the same spectrum and having the same distance from the solar system. We may possibly find suitable material in the group of the *Hyades*. Let the images of the brighter stars be artificially reduced by five or six magnitudes. The two sets of stars will behave as if they were of the same *apparent* magnitude. The distance, too, being the same, there is no other difference than that of the *absolute* magnitudes. We must get, therefore, a good determination of c quite independently of the other unknowns of the problem.

II. The value of c once found, the accurate determination of d becomes, relatively speaking, easy. Even the present investigations furnish a value, the accuracy of which, for a first determination, is not altogether unsatisfactory. There can be no doubt, however, that, from observations made expressly for the purpose, we may obtain better results. For

a) We may with advantage substitute photographic determinations of magnitude for visual measures. Whether objective-prism photographs of spectra for which the densities in the red and violet or ultra-violet could at once be compared, or photographs on red- and violet-sensitive plates are to be preferred, I am not prepared to decide. But it seems quite certain that photographic methods will lead to a considerable improvement.

b) By comparing parts of the spectrum sufficiently toward the extremes the quantities on which the whole determination rests may probably be much increased. If, for instance, we assume, as in the preceding article, that the difference *Phot.* − *Vis.* depends wholly

on the scattering of light, then by comparing the light at $\lambda = 0.65\ \mu$ and $\lambda = 0.35\ \mu$ the effect would be *more than fourfold* what it is in the difference *Draper Catalogue – Harvard Revision.*

c) We might base our observations exclusively on stars of widely different distances.

Notwithstanding all this, it seems probable that, in order to get really satisfactory results, the investigations must be extended over a large number of stars, so that a large amount of labor would be involved. I cannot help thinking, however, that the results would repay almost any amount of labor. In giving expression to this conviction, I am not considering the value of results of quite a different sort that such a spectro-photometric investigation is bound to yield. I take into account only the fact that the question of the amount of space-absorption must be faced, unless investigations concerning the real distribution of the stars in space are to come to a standstill. The admitted difficulty of the subject makes it imperatively necessary not to neglect any means that may contribute toward such a determination. Among these the method here followed seems to me one of the most promising, if not the most promising of all. But this is not all. In my first paper I ventured to call attention to the evident fact that a knowledge of the amount of selective absorption promises to yield data concerning distances which are beyond the power of other methods. Take, for instance, a cluster resembling the *Hyades* in spectrum and presenting somewhat the same structure. Let its apparent diameter be 5′ and let the brighter stars it contains reach the twelfth magnitude. Both by a comparison of the diameters and from the magnitude of the brightest stars we may estimate its distance to be of the order of fifty times that of the *Hyades*. The parallax of the *Hyades*[1] now being known with considerable accuracy to be 0″.024, the parallax of our cluster becomes of the order of 0″.00005.

It seems impossible to get a notion of the real distance of such a cluster in any of the ordinary ways. On the other hand with a parallax such as the above and a value of d of the magni-

[1] It has been determined by Boss and at Groningen by fundamentally different methods. The agreement of the results is very satisfactory. See *Astronomical Journal*, No. 604, and *Groningen Publications*, No. 23.

tude of that found in the present paper, the selective absorption would amount nearly to a *whole magnitude*. For regions in the ultra-violet this absorption would probably be considerably greater still. It need not be said that such an absorption must show at once in the spectrum. Its quantity could easily be determined with precision and, given a thoroughly reliable value of d, we would thus easily obtain a good estimate of the distance. The recent observations of Mr. Fath[1] might seem to confirm our expectations. But as we have only reproductions to judge by, it seems safer to wait for confirmation from the negatives and for results from more clusters.[2]

SUMMARY OF RESULTS

In the present state of the question the results of the present investigation are somewhat meager. They are as follows:

a) A relatively good determination has been obtained for the quantity $d-0.25c$. In consideration of what has been said in I*a* of the preceding section I adopt as final result the value (29)

$$d-0.25c=+0.00525\pm0.0010 \text{ Mag.} \tag{33}$$

b) The value of c, if not zero, must be small, but cannot be well determined from the present data. We found

$$c=+0.0058\pm0.0112 \text{ Mag.} \tag{34}$$

[1] *Lick Bulletin*, No. 149.

[2] While this paper was in the hands of the printer Mr. H. D. Babcock made a series of focal and extra-focal exposures with the 60-inch reflector on both ordinary and red-sensitive plates (the latter through a red screen). Defining the degree of redness as the difference in magnitude on the two kinds of plates, a provisional comparison of two clusters and the Great *Andromeda* Nebula, for all of which Mr. Fath has determined the type of spectrum, with stars of the same types of spectra gave the following results:

N. G. C. 7078	1^{m}.0	redder than a star of the same spectrum
N. G. C. 7089	0.45	" " " " " " " "
Andromeda Nebula	1.0	" " " " " " " "

In addition, the cluster in *Hercules* was found to be

+0^{m}.9	redder than a star of spectrum	F
−0.1	" " " " " "	K

These results are so strongly confirmatory of what the present investigation would lead us to expect, that the determination of the spectral type of other clusters and nebulae has been added to the working programme of the 60-inch reflector. Meanwhile another method of testing the existence of a selective space-absorption is being tried. Should this be successful it s hoped that all question as to the existence of an appreciable space-absorption and as to the order of its amount will be removed.

Corresponding with this we have for the value of d, independently of any assumption about the value of c,

$$d = +0.0066 \pm 0.0031 \text{ Mag.} \qquad (35)$$

c) There is no reason for assuming that the selective absorption is different for galactic and extra-galactic stars.

d) The values of *Phot.* − *Vis.*, freed from the influence of space-absorption, were found. These are given in Table XII.

MOUNT WILSON SOLAR OBSERVATORY
August 1909

Minor Contributions and Notes

ON COMPARISON PRISMS

In the May number of this *Journal* (**29**, 305, 1909), Dr. Bell describes several forms of comparison prisms said to have various advantages. He advocates in particular the two forms sketched in Figs. 2 and 5, on the ground that they should make it possible to illuminate the different portions of the slit of a spectrometer in such a way, for photometric purposes, that the spectra from the light-sources to be compared are separated by a practically invisible line of demarkation. If this could be attained, one of the forms suggested by Mr. Bell would, in fact, be a very useful accessory. It appears to me, however, and this is confirmed by experiment, that even with a long focal length of the collimator, the result sought for can be obtained only by the sacrifice of one of the essential conditions of spectrometric and spectral-photometric operations, namely, the complete illumination of the entire collimator lens from the light of the two sources. An examination of Bell's figures shows at once that for getting a sharp boundary the path of the two cones of rays must pass as near as possible to the limiting surfaces of the two halves of the prism, in order that the point of reflection of one of the two beams lie as near as possible in the plane of the slit. This, however, excludes the use of extended sources of light, as well as of condensing lenses of sufficient angular aperture. This deficiency could be removed at once by a small alteration of the prism for one of the two beams, as I probably need not describe in detail; but complications arise for the second beam, which at the same time require alterations at the head of the slit, the practical execution of which is at least doubtful.

H. Konen

Münster i. W., Universität
September 1909

ON THE SIZE OF METEORS

In his article with the above title, published in this *Journal* in June of this year (**29**, 378), Mr. William H. Pickering expresses the

opinion that the dimensions of shooting stars must be much larger than is ordinarily considered. He says:

> If the brilliancy of a meteor of the third magnitude was about that of the incandescent portion of an electric arc light, the meteor itself must be 6 or 7 inches (15 or 18 cm) in diameter. . . . . If, on the other hand, the meteor weighed only one grain, its intrinsic brilliancy must be 4000 times that of the incandescent carbons, or 40 times that of the sun itself.

These estimates appear to me to contradict the known photometric results. Several years ago I made comparisons between the luminous intensity of the stars and that of the standards employed in photometry.[1] I found that a star of magnitude zero gives an illumination of 2.1×10^{-6} *lux*,[2] when the star is at the zenith and the observer at the level of the sea, with good atmospheric conditions. We may readily deduce from this that a meteor of magnitude 3, distant 150 km from the observer, should have a luminous intensity of 3000 candles, if there were no atmospheric absorption; to allow for the absorption, let us call this 4500 candles. On the other hand, the crater of the electric arc gives about 200 candles per square millimeter. The projection of the visible surface of the meteor, supposing that it has an intrinsic brightness equal to that of the crater of the arc, would be 23 square millimeters. Supposing it spherical, its diameter would be 25.4 mm, and if its density was 4, it would weigh 0.3 grams (4.5 grains).

see Page 400

It is very difficult to discuss the correctness of the hypothesis that the intrinsic brightness is equal to that of the electric arc. Mr. Pickering regards that value as a maximum, and it is strongly probable that it may be so. It is necessary to remark further that the mechanism of the incandescence should be very nearly the same in the case of the arc and that of a meteor: a bombardment of a solid body by corpuscles having a very great relative velocity (ions in the one case; gaseous molecules in the other), and distant enough from each other so that they behave like separate projectiles. In the electric arc, certain metallic oxides give a more brilliant crater than

[1] "Recherches de photometrie solaire et stellaire," *Association Française pour l'Avancement des Sciences*, 1903.

[2] The *lux*, the unit of luminosity, is the amount of light produced by a candle on a screen at a distance of one meter.

that of carbon. It appears to me impossible to say how the matter in the meteors would behave in this respect. It seems that if their temperature was considerably lower than that of the arc, their light ought to appear strongly reddish.

CH. FABRY

UNIVERSITÉ DE MARSEILLE
October 1909

CORRECTION

I am indebted to Dr. H. Ludendorff for the following corrections to my note on the stars in *Ursa Major* (this volume, page 135):

1. For δ *Leonis* (Table I) $\mu \sin (O.-C.) = +0''.0128$ (not $0''.0032$) yearly.

2. The calculated parallax of β *Aurigae* should be $0''.024$ (not $0''.030$) and consequently $x=7.8$, $y=-18.8$, $z=36.3$, and the calculated absolute brightness -6.03 mag. instead of -5.53 mag.

E. HERTZSPRUNG

GÖTTINGEN
October 12, 1909

Reviews

Formeln und Hilfstafeln für geographische Ortsbestimmung. Von Th. Albrecht. Vierte Auflage. Leipzig: Wilhelm Engelmann, 1908. 28×20 cm. Pp. 348. M. 20; bound, M. 22.

The three previous editions of this work have been favorably known to many outside of the smaller circle to whom the title in strictness would directly appeal. The new edition is very considerably enlarged and improved, and is far more than its name implies: only the second half of it is taken up with the tables, and the first half is a concise treatise on the topics considered, such as the relation between spherical co-ordinates for special cases, the determination of time, latitude, and azimuth, the telegraphic determination of longitude.

An appendix at this point gives the formulae dealing with the figure of the earth, and related matter.

The tables are printed in clear type and in a style which has borne the test of practical experience; they are 47 in number, and include all that could be expected in such a work, and many not found in such collections. Tables 31*a* to 31*g* give Gyldén's refractions, corrected to accord with the latest data. Tables 32*a* to 32*l* deal with the figure of the earth, including Bessel's tables for transformation to the spheroid, also those of Schreiber. Table 45 gives four-place logarithms of the trigonometric functions with argument in time useful in reduction to apparent place. The book can be recommended to all interested in practical astronomy and geodesy.

F.

Tafeln für numerisches Rechnen mit Maschinen. Herausgegeben von O. Lohse. Leipzig: Wilhelm Engelmann, 1909. Pp. 122. M. 12; bound, M. 13.50.

These tables conform in size closely to those of Albrecht, just mentioned. The first twelve pages give the reciprocals of all numbers up to 5000, a special convenience in making it possible to use multiplication instead of division with calculating machines. The next 90 pages contain the six natural trigonometric functions to hundredths of degrees; then follow tables of squares to 1000, and of square-roots from 1000 down. A collection of trigonometric formulae is also attached, with conversion-

tables for time into angles, etc. The eighth and last table gives the wavelengths of 115 spectral lines, almost all due to iron, by Fabry and Buisson. The book will be of useful service to practical astronomers and geometers.

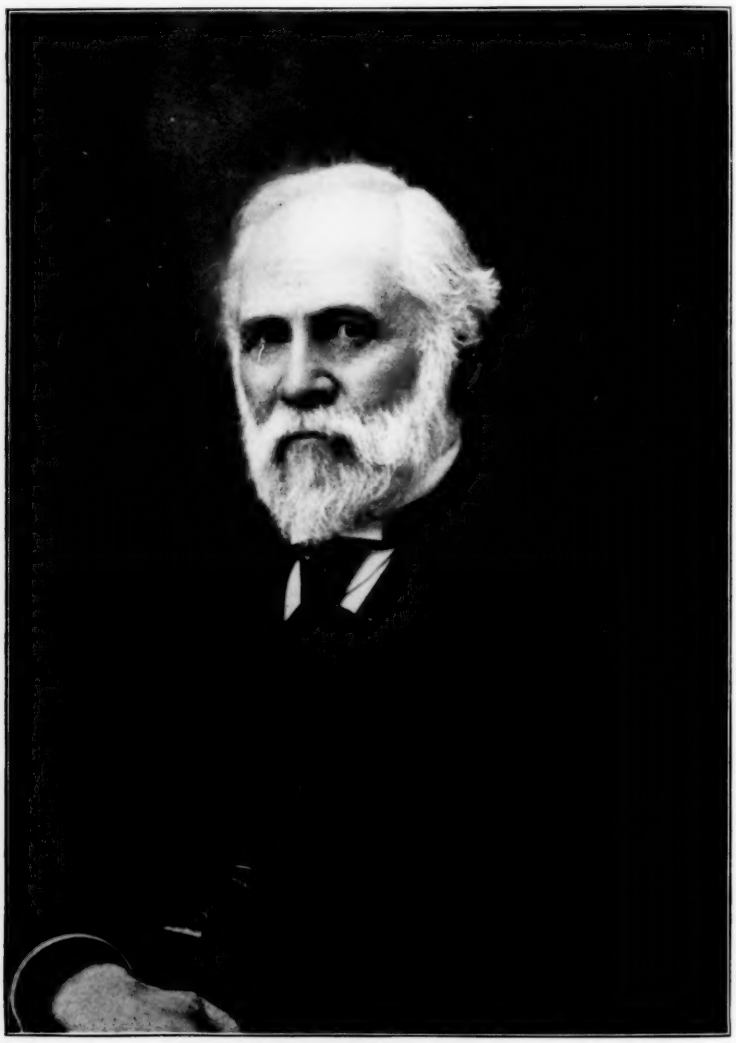
F.

Astronomisches Jahresbericht. X. Band, Die Literatur des Jahres 1908. Herausgegeben von A. BERBERICH. Berlin: Georg Reimer, 1909. Pp. 746. M. 23.

This annual, published under the auspices of the Astronomische Gesellschaft, aims to give a brief summary of every astronomical paper that has appeared in the calendar year. It has become an indispensable tool for the working astronomer. Save for two or three minor changes the current volume follows the general arrangement of its predecessors. The number of papers reviewed is smaller than in any other volume since the first. On the other hand the number of pages is greater than ever before, so that the average length of the reviews has greatly increased. The reviews are intended to be mere summaries and not to be critical in character, but the editor has succumbed to the temptation of inserting here and there a very expressive exclamation point.

The standard of accuracy set for the *Jahresbericht* by its founder, the late Dr. Wislicenus, was very high. A somewhat superficial examination has given the present writer the impression that the volume before us falls a little below in this respect. The truth is that this undertaking is greater than the energies of one man can meet; for it is a fact that, in spite of a somewhat formidable list of collaborators, almost all the summaries have been made by the editor himself. It is no small task to read carefully 1500 articles published in at least five different languages and treating of the great variety of subjects with which astronomers are now concerned. Would it not be helpful if authors would prepare summaries of their own papers and forward them promptly to the editor? The latter could, whenever he deemed it advisable, cut these down to appropriate size. Doubtless if the editor would let it be known that these summaries would be acceptable, such a request would be quite generally complied with by astronomers.

FRANK SCHLESINGER



CHARLES AUGUSTUS YOUNG